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REDUNDANT SENSOR STRAPDOWN IMU EVALUATION
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GENERAL TEST PLAN
REDUNDANT SENSOR STRAPDOWN IMU
EVALUATION PROGRAM

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Prepared for

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TABLES

ILLUSTRATIONS

1. INTRODUCTION

This General Test Plan is submitted to NASA MSFC as the second major deliverable on Contract NAS 8-27335. The plan defines the test series necessary for the completion by MSFC of a redundant sensor strapdown IMU evaluation experiment.

1.1 EXPERIMENT OBJECTIVES

This experiment series uses a prototype (breadboard) strapdown IMU. It contains redundant gyros (six in a dodecahedron), and three orthogonal accelerometers. The system software incorporates failure detection and correction logic and a land vehicle navigation program.

A significant feature of this experimental program is the use of Single Axis Reference units (commonly called single axis platforms) for attitude reference. This device is described in Reference 1. Hardware experimentation with redundant strapdown systems using conventional single degree of freedom gyros has been done elsewhere (MIT, ERC).

The principal objective is a demonstration of the practicability, reliability, and performance of a SAR redundant strapdown IMU system with failure detection and correction, in operational environments.

Engineering information will be obtained on permissible failure threshold settings with real instruments and the accumulation of navigation errors before a failure is detected.

1.2 SCOPE

This plan includes:

- Equipment Configuration
- Error Analysis
- Software Requirements
- Test Descriptions

Configuration

The equipment configuration and software requirements were established in the Program Definition and Experiment Configuration Plan, Reference 2, and only a summary and revisions are included in this plan.

Error Analysis

Design of the system tests and analysis of the test results requires analysis and modeling of system errors. A preliminary error analysis of the MSFC Breadboard Dodecahedron SAR Cruise Navigation system has been included in this plan. The analysis is in sufficient depth to allow:

1) evaluation of calibration and compensation requirements

2) test design (sequence, input parameters, duration)

A continued error analysis effort will be required by MSFC for interpretation of test results, setting performance criteria, and refinement of the test sequence.

Test Plans

The test objectives, constraints, description and analysis (Section 5 through 9) are divided into five test phases, as shown in Figure 1-1. The test descriptions are based on the assumption that the hardware and software configurations described in Sections 2 and 4 have been implemented and completely checked out. The test sequence is arranged such that the calibration, alignment, and navigation functions are completely checked out prior to exercising the FDDC logic with failures, simulated failures, and degraded operation.

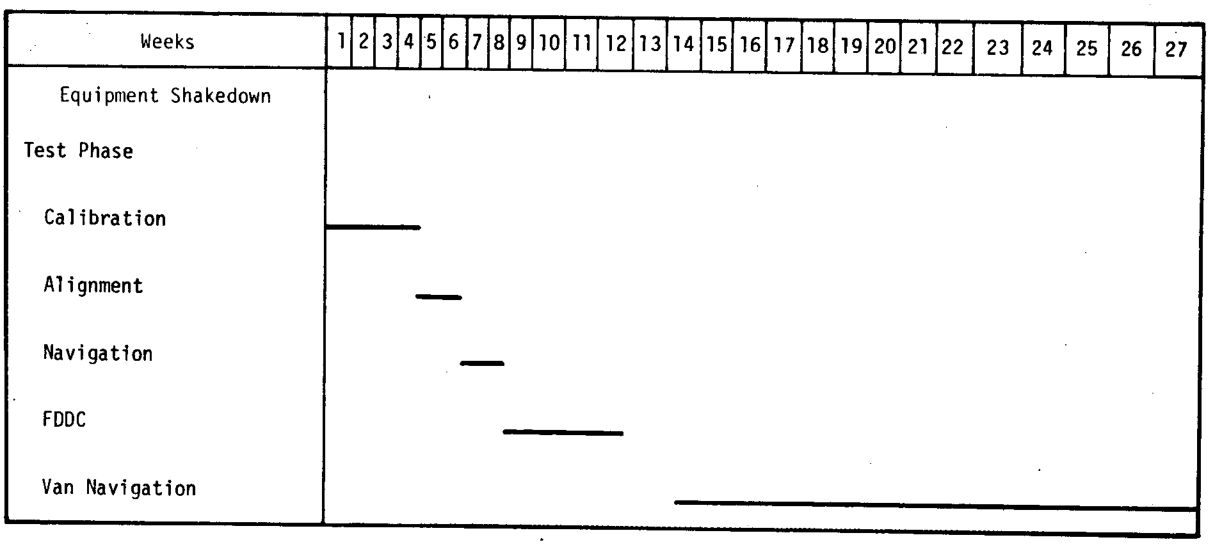


Figure 1-1. Overall Test Sequence

2. EQUIPMENT CONFIGURATION

The configuration of the test article, test station, and mobile van upon which the test series is based are documented in Reference 2. Only changes and additions are listed below.

2.1 BREADBOARD DODECAHEDRON (BB DDH)

The accelerometer torquer loops are being modified to reduce the quantization from .4 Fps/pulse to .15 Fps/pulse. This will reduce the alignment filter convergence time and reduce the noise in the calibration and navigation routines.

The SAR which did not have a gimbal reference position output pulse is being reworked. All six SAR's will have a zero reference pulse.

Modifications are being made to the BB DDH electronics package to:

1) Combine positive and negative output pulses from each sensor onto a single output line to reduce the number of recording channels required in the van.

2) Combine three fault monitors (wheel current, gimbal spin detector, and encoder error) into a single failure discrete.

These changes are shown in the revised BB DDH block diagram, Figure 2-1.

Another change from Reference 2, Paragraph 2.3.4 is; the three torquing rates available from the Control Panel (Figure 2-1) are:

0.4 degree/hour
4.0 degree/hour
60 degree/minute

2.2 MOBILE VAN CONFIGURATION

The van instrumentation is being developed approximately as described in Reference 2, Section 2.4 and repeated in Figure 2-2, although the design is not yet complete. The alternate configuration employing two tape recorders in the van will not be used. Redstone has the IRIG B time code on a radio link and a receiver may be installed in the van. The van will be air conditioned. Van instrumentation is discussed in Paragraph 9.4.2.

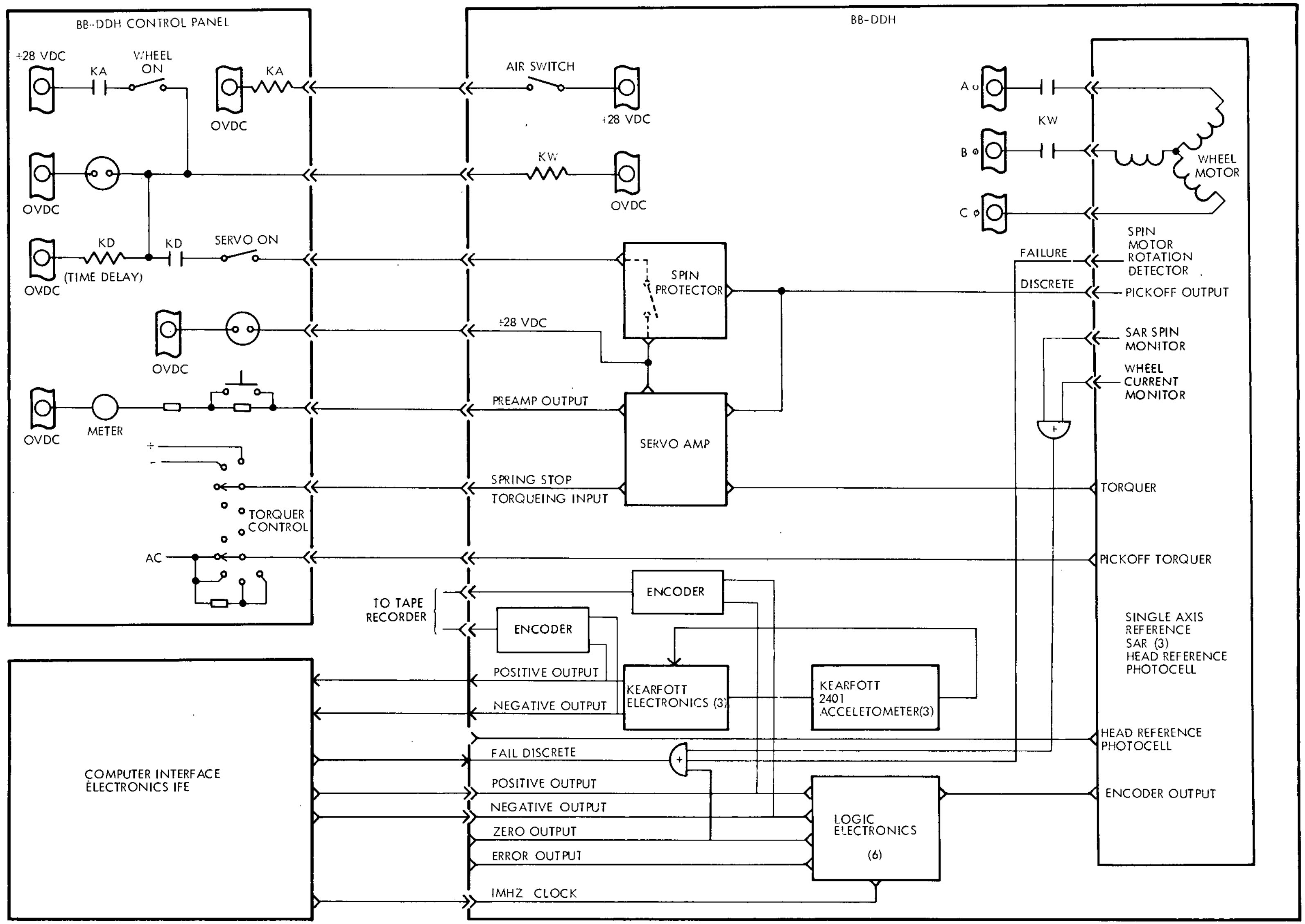


Figure 2-1. Block Diagram BB DDH, Control Panel, and IFE Interfaces (Preliminary)

2.3 INTERFACE DEFINITION

The equipment interfaces are as defined in Reference 2, Section 3, except for the change in accelerometer quantization previously discussed.

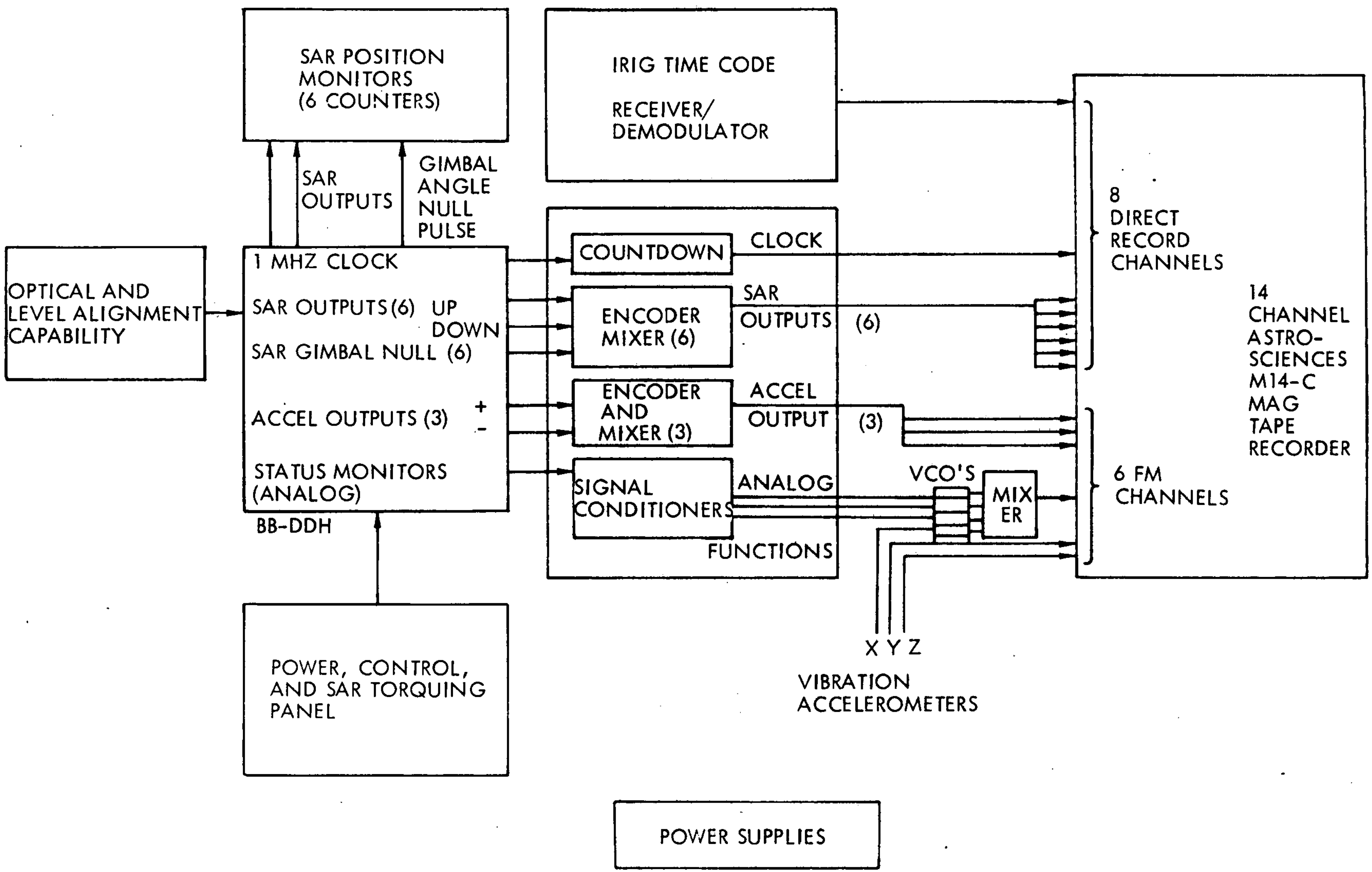


Figure 2-2. Mobile Van Functional Block Diagram (Conceptual)

3. ERROR ANALYSIS

A preliminary analysis of navigation errors in the laboratory and van tests is presented in Appendix A. In this analysis, the navigation error equations of a nonredundant strapdown IMU are described. This description is then extended to a dodecahedron with no failures. Finally, the performance degradation associated with failure detection, diagnosis and correction (FDDC) is discussed.

A preliminary navigation error estimate for the BB DDH is also presented. Performance is estimated both without and with a gyro drift update prior to navigation. Also, performance is estimated both without and with a redundant sensor program (RSP) change to compensate gyro mass unbalance and SAR misalignment as a function of SAR gimbal angle.

In Appendix A, attitude and navigation error sensitivities and navigation error estimates are derived in Customary Units. These error sensitivities and error estimates are presented in Figures 3-1 through 3-9 and in Table 3-1 in both Customary Units and in the International System of Units.

3.1 NONREDUNDANT STRAPDOWN IMU

The attitude, velocity and position errors that result from the following error sources in a nonredundant strapdown IMU are described in Appendix A.

- initial azimuth alignment error
- initial level alignment error
- level gyro drift
- vertical gyro drift
- gyro misalignment
- level accelerometer bias error
- level accelerometer scale factor error
- level accelerometer misalignment

Error correlations involving initial azimuth alignment error and level gyro drift, and initial level alignment error and level accelerometer bias error are also described.

It is shown in Appendix A that the attitude and navigation errors that result from an initial azimuth alignment error are of the same form as the corresponding errors resulting from level gyro drift. Similarly, the attitude and navigation errors that result from an initial level alignment error are of the same form as corresponding errors resulting from level gyro drift, vertical gyro drift and level accelerometer bias error. The latter statement is also true for gyro misalignment and level accelerometer misalignment. Thus, with the exception of the attitude and navigation errors that result from level accelerometer scale factor error (and with the exception of certain other second order errors), all attitude and navigation errors are of the same form as those resulting from level gyro drift, vertical gyro drift and level accelerometer bias error. The latter errors are plotted in Figures 3-1 through 3-9.

It is shown in Appendix A that the attitude and navigation errors resulting from level accelerometer scale factor error will be much less than corresponding errors resulting from other error sources. Hence, level accelerometer scale factor errors may be ignored in this analysis.

3.2 STRAPDOWN DODECAHEDRON

The sensor error coefficients of a strapdown dodecahedron may be related to the corresponding error coefficients of a nonredundant IMU.

Assume that the gyro bias errors of a dodecahedron are statistically independent with zero mean and equal variance. It is shown in Appendix A that for no failures,

$$\begin{bmatrix} \sigma_{b_x} \\ \sigma_{b_y} \\ \sigma_{b_y} \end{bmatrix} = \sigma_B \begin{bmatrix} .707 \\ .707 \\ .707 \end{bmatrix}$$

where σ_b^2 = gyro bias variance

$\sigma_{b_i}^2$ = equivalent i^{th} axis variance of a nonredundant IMU

Similarly, for gyro spin axis mass unbalance and assuming that the level acceleration input components are negligible,

$$\begin{bmatrix} \sigma_{b_x} \\ \sigma_{b_y} \\ \sigma_{b_z} \end{bmatrix} = \sigma_s \begin{bmatrix} .548 \\ .316 \\ .316 \end{bmatrix}$$

where σ_s^2 = gyro spin axis mass unbalance variance

and the x axis is vertical.

For gyro input axis mass unbalance and assuming that the SAR gimbal angles as defined in Appendix A are equal to 90 degrees,

$$\begin{bmatrix} \sigma_{b_x} \\ \sigma_{b_y} \\ \sigma_{b_z} \end{bmatrix} = \sigma_I \begin{bmatrix} .447 \\ .548 \\ .548 \end{bmatrix}$$

where σ_I^2 = gyro input axis mass unbalance variance

For the SAR misalignments, assuming that the level angular rate input components are negligible and ignoring the internal misalignments,

$$\begin{bmatrix} \sigma_{b_x} \\ \sigma_{b_y} \\ \sigma_{b_z} \end{bmatrix} = \sigma_c \begin{bmatrix} .447 \\ .548 \\ .548 \end{bmatrix}$$

where σ_c^2 = SAR misalignment variance.

Equations extending the foregoing relations to include the effects of SAR gimbal angle changes are provided in Appendix A.

3.3 FAILURE DETECTION, DIAGNOSIS AND CORRECTION

A failure of one or more sensors in a dodecahedron will cause system performance to be degraded. Degradation may result from one or more of the following:

- undetected errors
- transient errors
- errors resulting from the deletion of sensors in the estimation equations

Each of the foregoing sources of error may be evaluated with the RSP in the Simulation Mode and with the error equations presented in Appendix A.

3.4 ESTIMATED PERFORMANCE

A rudimentary estimate of navigation accuracy in the forthcoming tests is presented in Appendix A. This estimate is based on the estimated BB DDH performance characteristics provided by MSFC and summarized in Table A-1. The navigation accuracy estimate is summarized in Table 3-1.

It is evident from Table 3-1 that navigation accuracy will be improved by the addition of a gyro drift update prior to navigation and by an RSP change to compensate gyro mass unbalance and SAR misalignment as a function of SAR gimbal angle. It appears, however, that the objectives of the current test series can be realized without RSP changes. (If necessary, a work around procedure can be devised to provide a gyro drift update prior to navigation). Hence, it is recommended that RSP changes to improve navigation accuracy be deferred.

Table 3-I. Navigation Accuracy Estimate

Test Conditions			Estimated Accuracy	
DDH rotated during navigation	Gyro drift update prior to navigation	RSP change to compensate gyro mass unbalance and SAR misalignment as a function of SAR gimbal angle	n.mi./hr	Km/hr
No	No	No	4.6	8.5
No	Yes	No	1.3	2.4
Yes	No	No	9.1	17.0
Yes	Yes	No	5.1	9.4
Yes	Yes	Yes	1.9	3.5

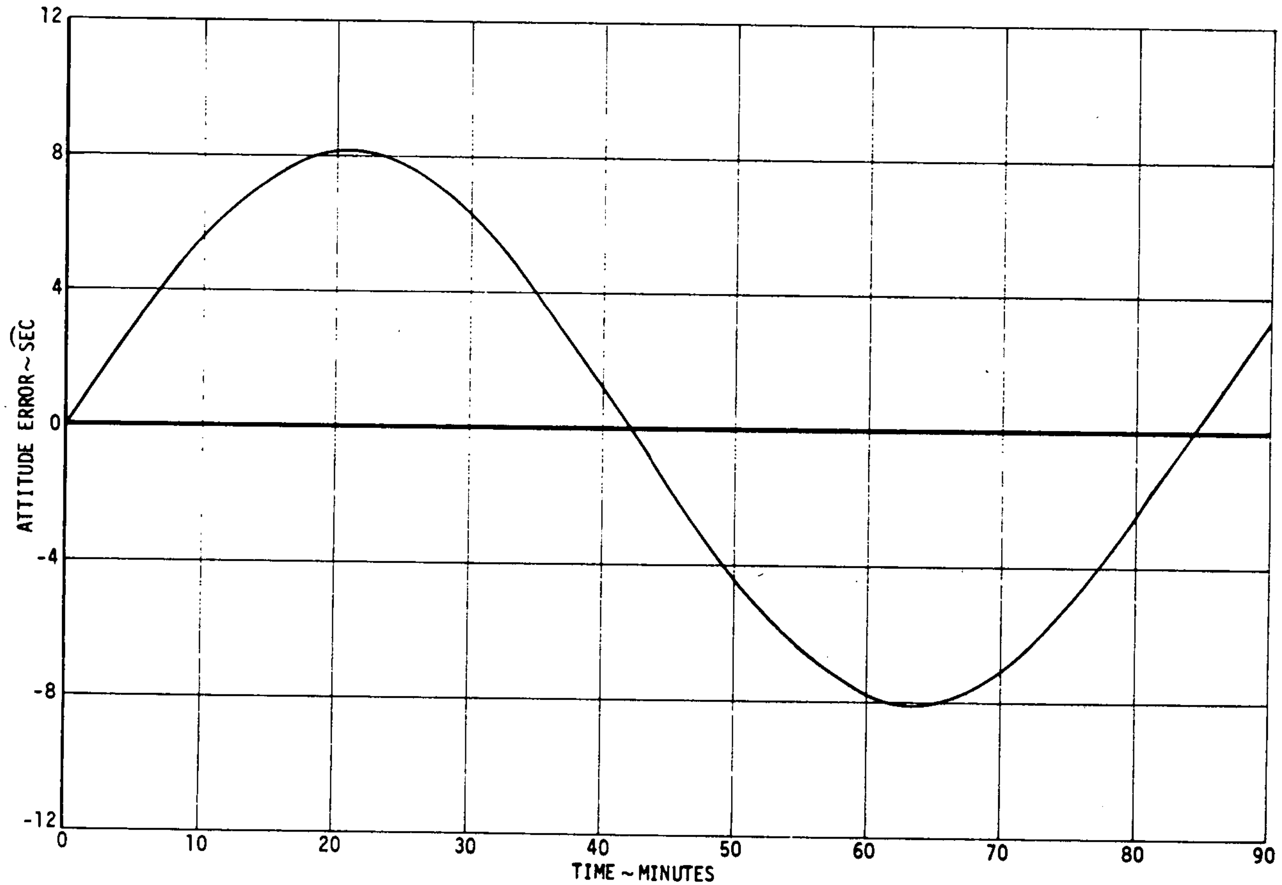


Figure 3-1. Attitude Error versus Time Level Gyro Drift — 0.01 deg/hr

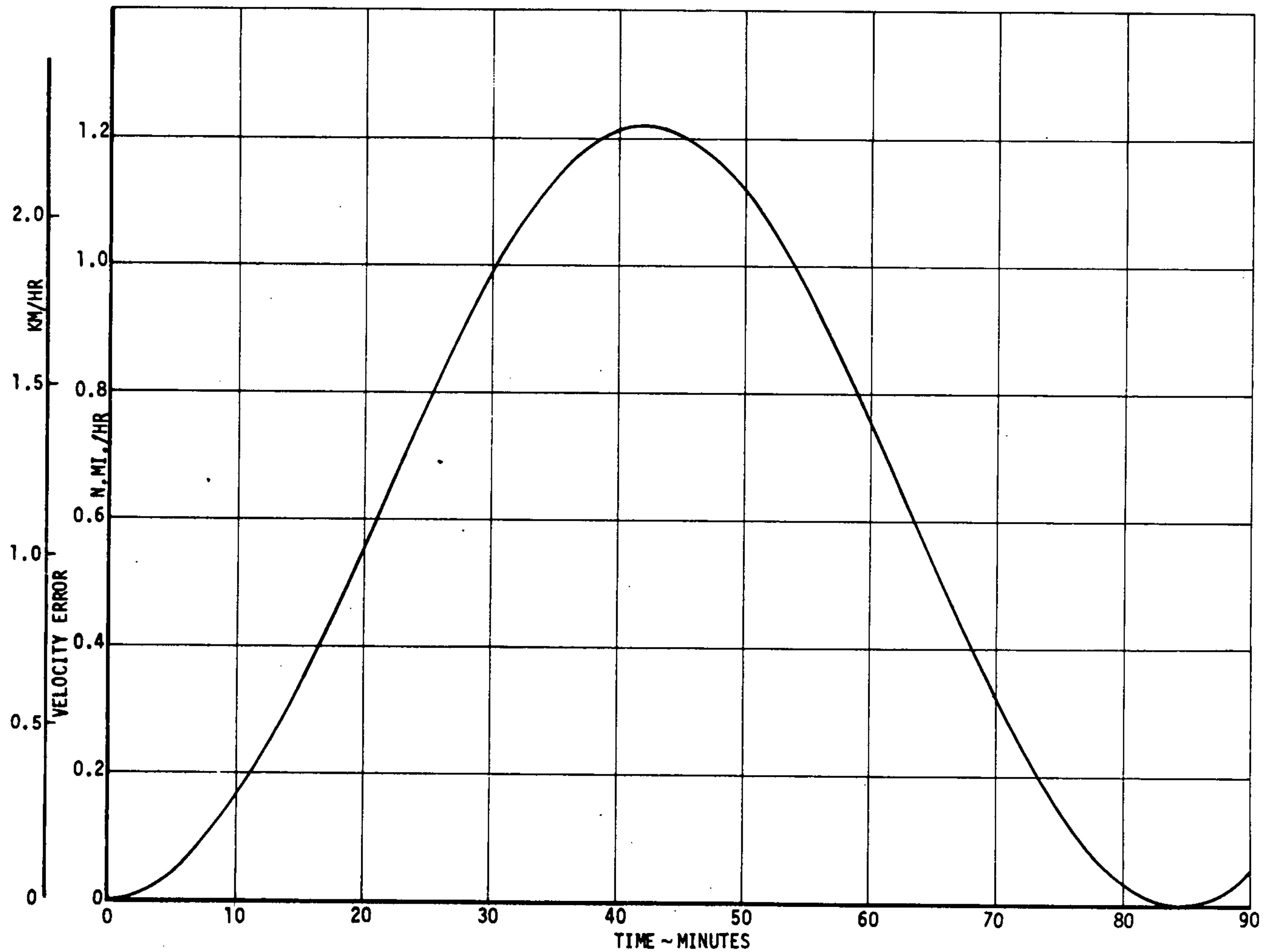


Figure 3-2. Velocity Error versus Time Level Gyro Drift — 0.01 deg/hr

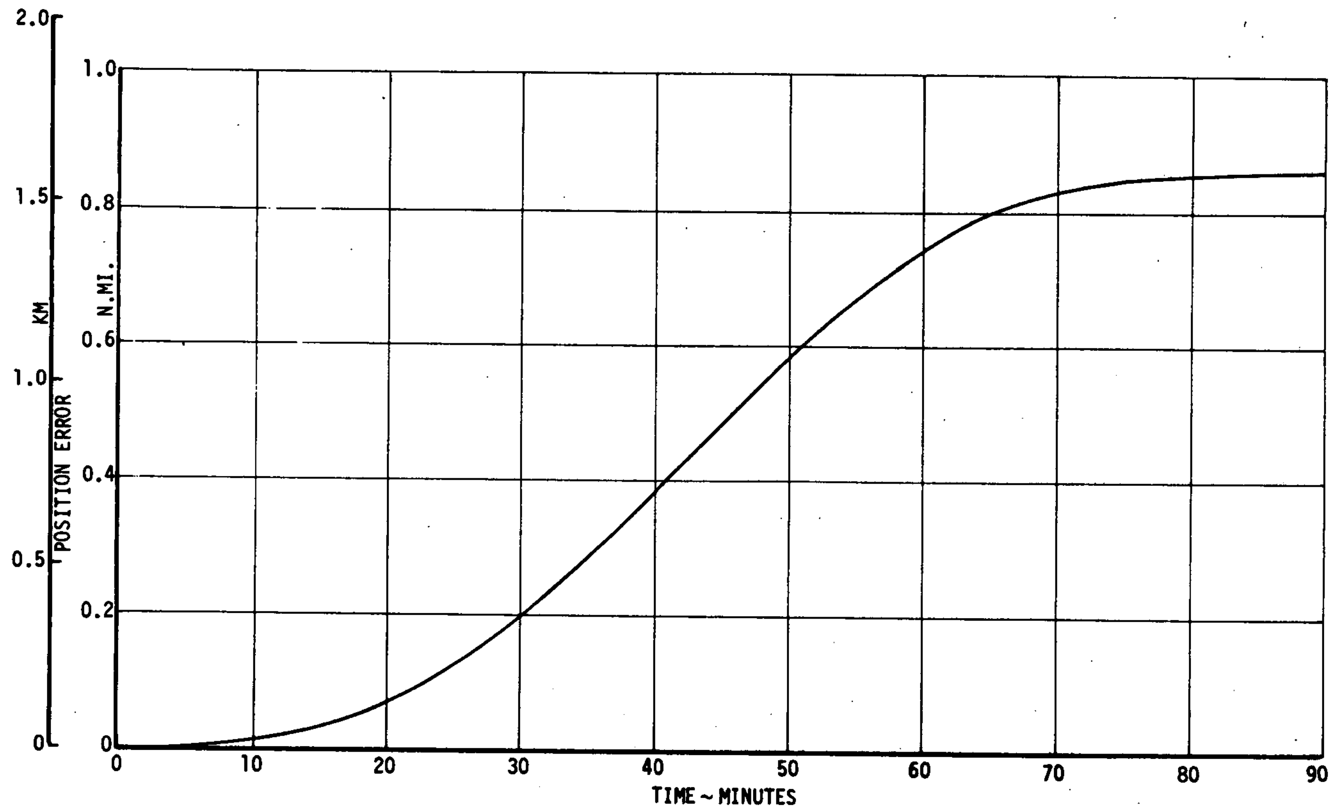


Figure 3-3. Position Error versus Time Level Gyro Drift — 0.01 deg/hr

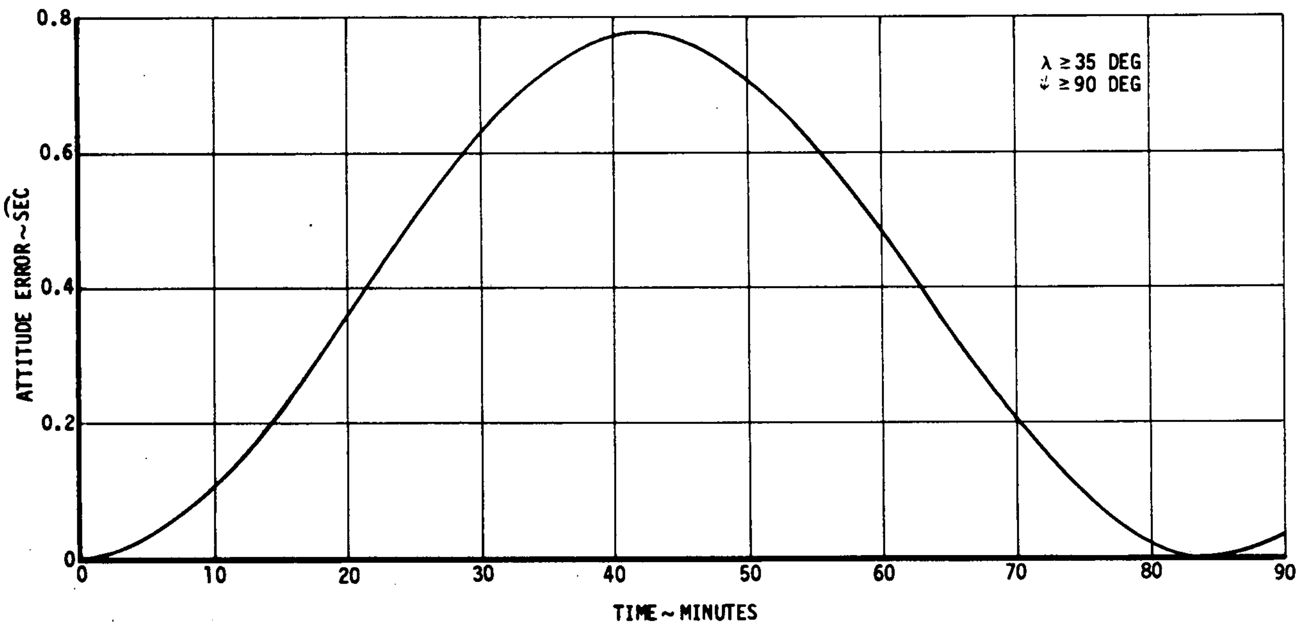


Figure 3-4. Attitude Error versus Time Vertical Gyro Drift — 0.01 deg/hr

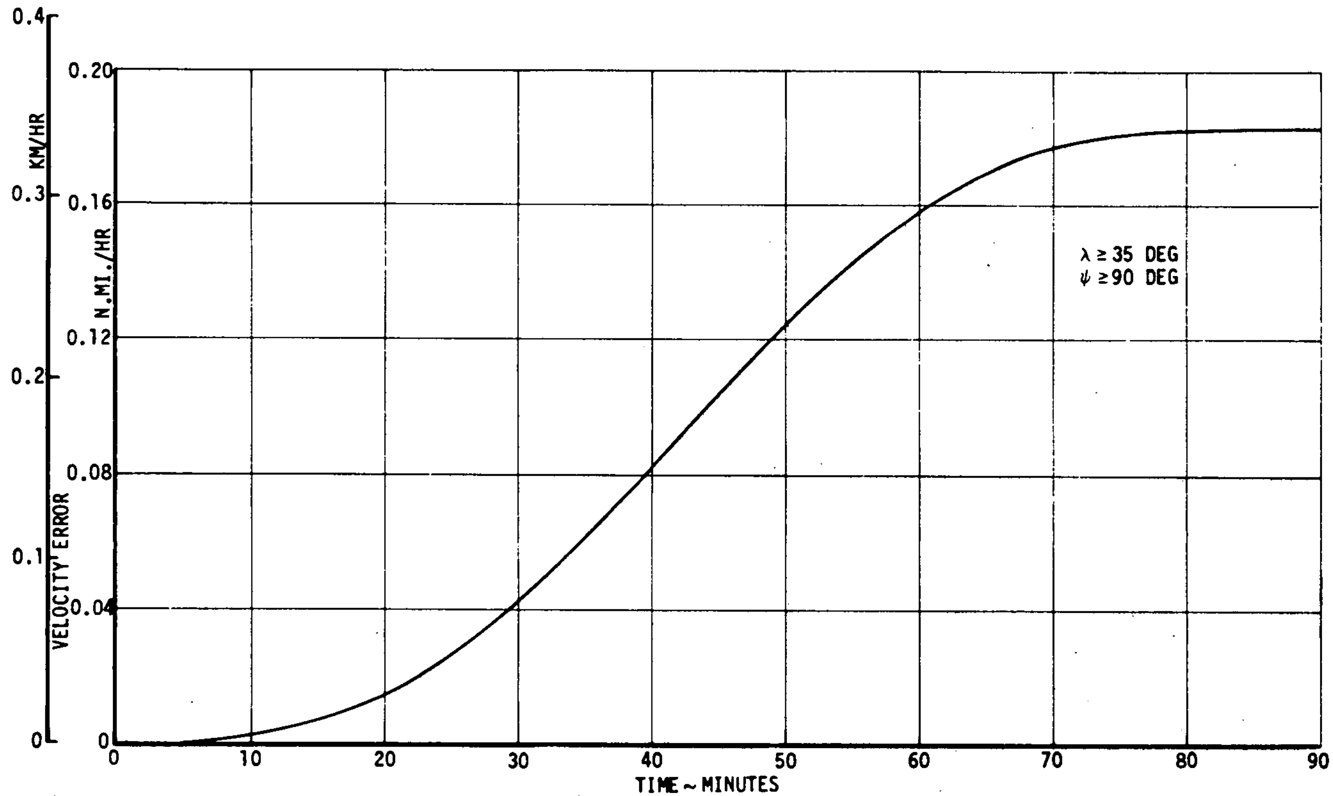


Figure 3-5. Velocity Error versus Time Vertical Gyro Drift — 0.01 deg/hr

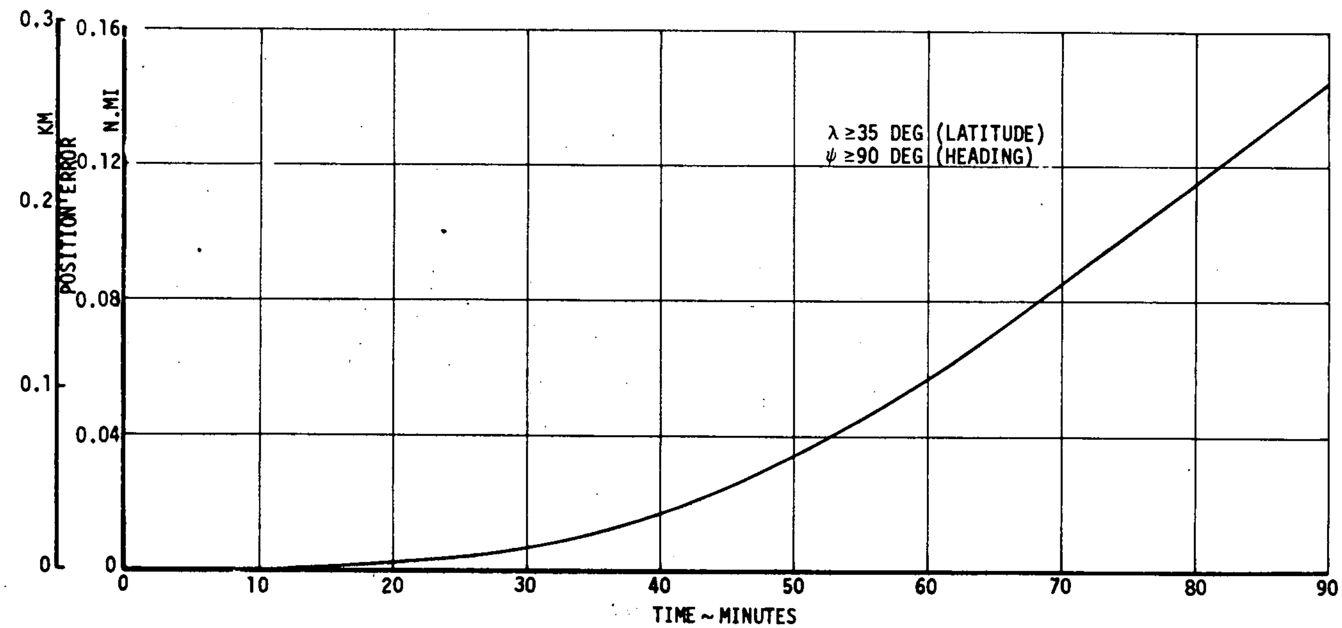


Figure 3-6. Position Error versus Time Vertical Gyro Drift — 0.01 deg/hr

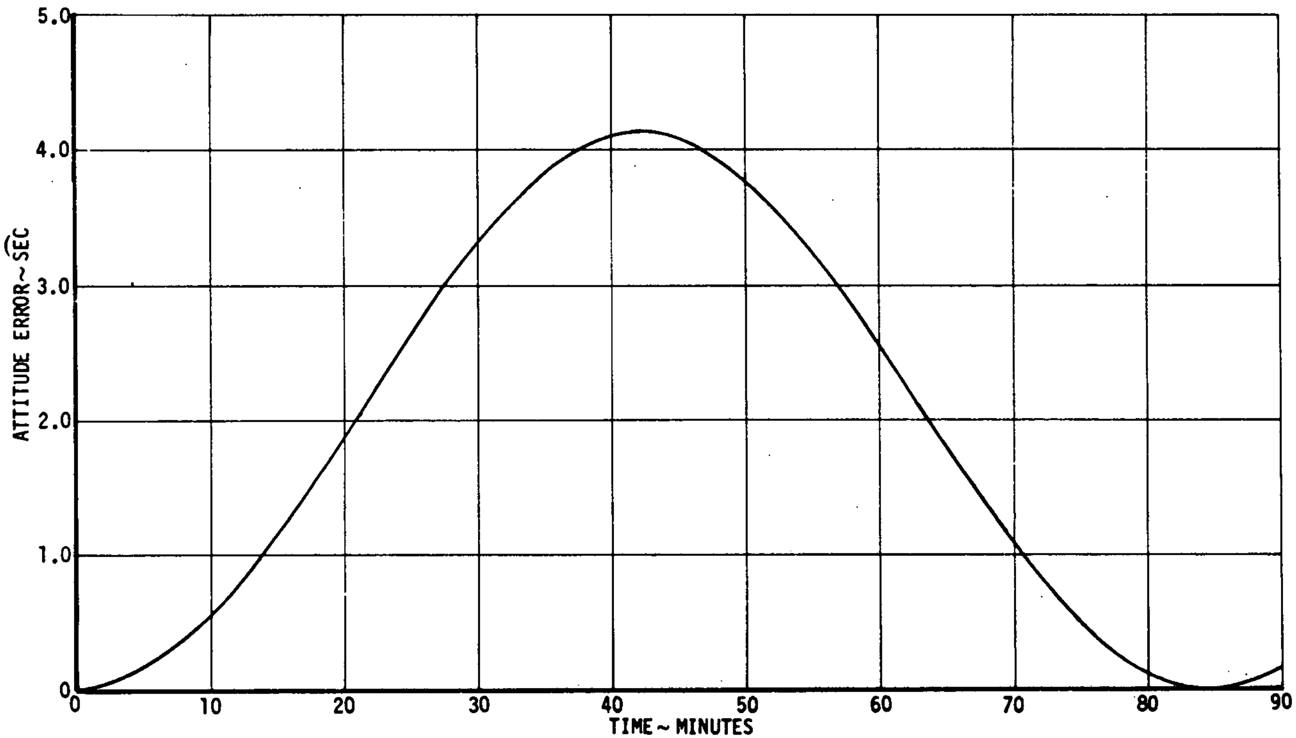


Figure 3-7. Attitude Error versus Time Level Accelerometer Bias — 10 μg

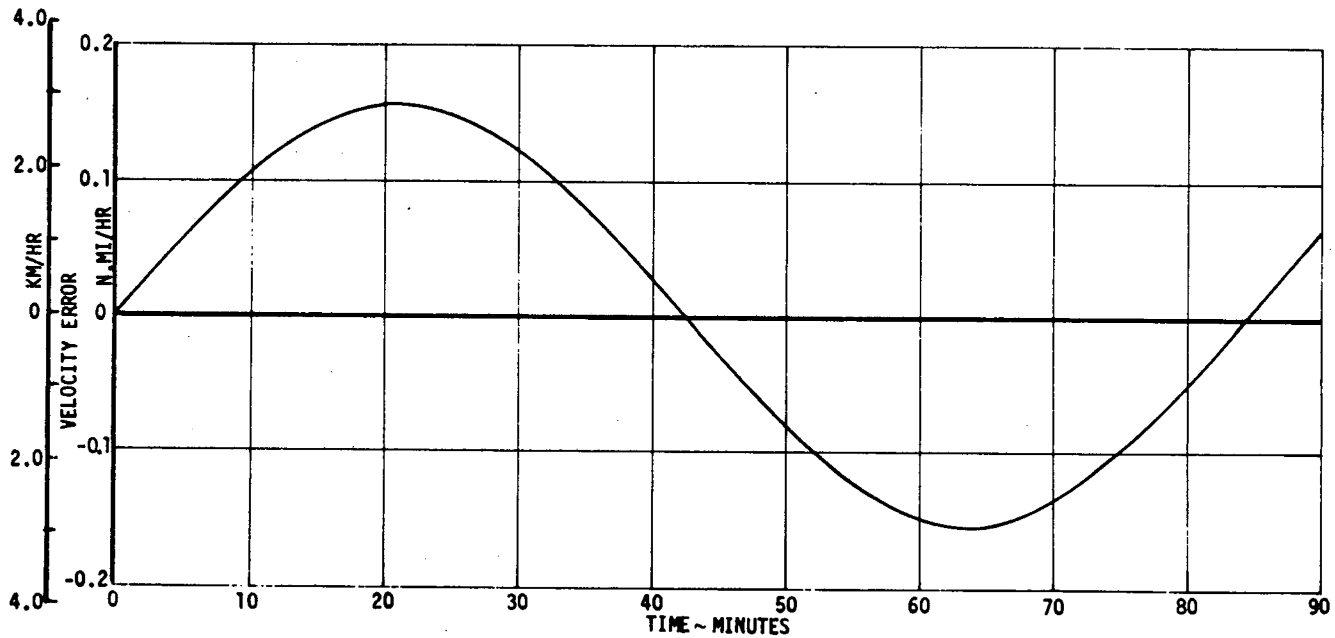


Figure 3-8. Velocity Error versus Time Level Accelerometer Bias — 10 μg

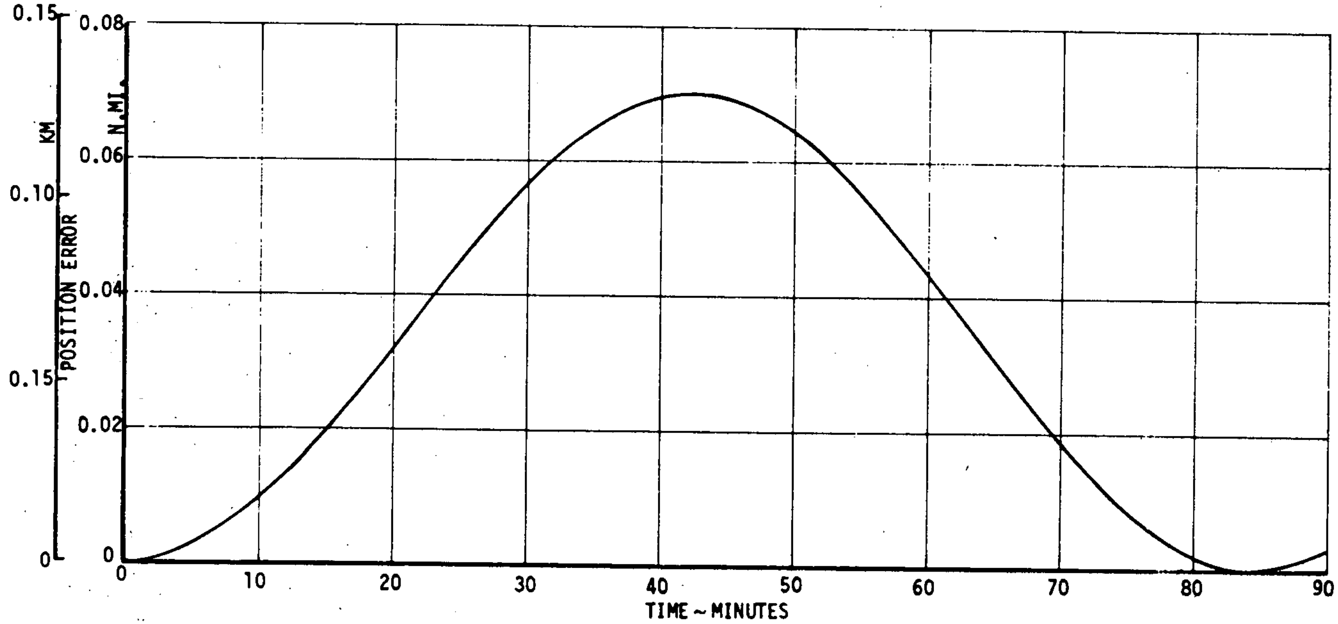


Figure 3-9. Position Error versus Time Level Accelerometer Bias — 10 μg

4. SOFTWARE CONFIGURATION AND CHECKOUT

4.1 PROGRAM MODIFICATIONS

Reference 2 defined the Redundant Sensor Program (RSP) revisions resulting from the changes in configuration between ERC and MSFC experiments. These are differences in:

- dodecahedron configuration
- test equipment configuration
- test site location

In this section all mandatory changes are tabulated and optional changes are identified. Section 3, Error Analysis, establishes the approximate system error attributable to the optional changes. Recommendations are made for changes to be incorporated, and the test plans are based on these recommendations.

4.1.1 Calibration Program

4.1.1.1 Instrument Change

	Discussion Reference 2 Section No.	Page
Nominal Gyro Bias	4.2.2	4-3
Nominal Accelerometer Bias	4.2.2	4-3
Gyro Scale Factor	No change	
Accelerometer Scale Factor	4.2.2	4-4

4.1.1.2 Site Dependent Constants

Latitude λ	4.2.2	4-4
Azimuth of Assembly Axes Θ	4.2.2	4-4

4.1.1.3 Static Test Interval Change — Note 1

(from 100 seconds to 240 seconds)

4.1.2 Alignment

4.1.2.1 Instrument Change

	Section No.	Page
SIGA (quantization noise)	4.2.3	4-5
SIG1,2,3 (initial misalignment estimate)	4.2.3	4-5

4.1.2.2 Site Dependent	Section No.	Page
AZHT (initial azimuth estimate)	4.2.3	4-6
RILX (inertial to VEN transformation)	Note 2	
4.1.2.3 Real Time Alignment Error Output	Note 3	
4.1.3 Navigation and FDDC		
4.1.3.1 Instrument Change		
FDDC		
• Bypass Accelerometer C matrix	4.2.2	4-5
• Bypass Accelerometer FDDC	4.2.2	4-5
• Accelerometer State Flag (Sa)	4.2.2	4-5
• Accelerometer Nominal A matrix	See Note 4	
• Gyro Nominal A matrix	See Note 4	
• Gyro Nominal C matrix	See Note 4	
• Addition of Failure Discrete Input Processing	See Note 5	
Navigation		
• Sensor quantization change	4.2.4	4-8
• Sensor nominal bias	4.2.4	4-8
4.1.3.2 Site Dependent Constants		
Latitude, Longitude, Gravity	4.2.4	4-7
4.1.4 Driver Routine		
• Sensor quantization, QA, QG	4.2.4	4-7,8
• Sensor nominal bias	4.2.4	4-8
4.1.5 Display		
Nixie Display to HP-2116 CRT Display	4.2.6	

Note 1:

To change the nominal static test calibration interval from 100 seconds to 240 seconds, change the input parameter STLT, static test sampling time in milliseconds to 240,000 (B23). (See Page 4-4, Reference 2).

Note 2:

Computation of the new R_I^L matrix is explained on Page A-13, Volume 1, Reference 3.

Note 3:

Alignment

If the parameters computed in Figure B63 of Volume 2, Reference 4 are desired for a real time alignment mode, the following changes should be made.

1) After the sequence number 4978 (JPL A13), insert the following instructions:

JMP PD6

2) Change the following instructions:

Sequence No.	Was	Change To
4981	JMP PD6	JMP D00
3919	JMP D00	JMP A11+5

The "K" matrix (3x3) must be initialized to the proper value at the beginning of a run.

Note 4:

1) Accelerometer A Matrix

$$\begin{array}{c|ccc} & X & Y & Z \\ \hline Z & 0 & 0 & 1 \\ & 0 & 0 & 0 \\ X & 1 & 0 & 0 \\ & 0 & 0 & 0 \\ Y & 0 & 1 & 0 \end{array}$$

2)

$$\begin{array}{c|ccc} & X & Y & Z \\ \hline Z_1 & b & o & a \\ Z_2 & b & o & -a \\ X_2 & a & -b & o \\ X_1 & a & b & o \\ Y_1 & o & a & b \\ Y_2 & o & a & -b \end{array}$$

b = .52573111
a = .8506508

Note 4: (continued)

3) Gyro C Matrix (15 x 6)

o	o	-a	a	-b	-b
o	-a	o	b	-a	b
o	-a	b	o	-b	a
o	b	-a	b	o	-a
o	-b	-b	a	-a	o
a	o	o	-b	-b	a
a	o	-b	o	-a	b
-b	o	-b	a	o	-a
b	o	-a	b	-a	o
-b	b	o	o	a	-a
a	b	o	-a	o	b
-b	-a	o	a	-b	o
-b	-a	a	o	o	b
a	b	-a	o	-b	o
a	a	-b	-b	o	o

Note 5:

Failure Discrete Input Processing

In order for FDDC to do real time discrete monitoring of the six gyros, some minor program changes will be required. These changes and the coding that already exists in FDDC and the real time input routine are based on the following assumptions:

a) An 18-word list will be read in every 40MS from the IFE.

b) The first six words correspond to gyros one through six.

c) The accelerometer pulses are in words seven, nine, and eleven.

d) The table angle, scaled at +15, is in word 13.

e) Gyro monitor discrete is in word 14.

f) The format of word 14 is as shown below:

BIT POSITION

0 1 2 3 4 5	6 7 8 9	10 11	12 13 14	15 16 17	18 19 20	21 22 22
not used	Gyro 6	Gyro 5	Gyro 4	Gyro 4	Gyro 2	Gyro 1

GYRO DISCRETE INPUTS

Note 5: (continued)

The low order 18 bits are used to store the gyro discrete data. Three bits are used for each gyro and if any one or more of the bits are on (equal to 1) FDDC will treat that gyro as having failed until all three bits are ZERO.

If the following modifications are implemented, then word 14 will be saved and used by FDDC for gyro internal discrete monitoring.

Replace the instruction JMP FO at sequence number 6596 with the following:

LDA BUF1 + 13	Gyro Discrete word
STA FFDG	FDDC Monitor word for gyros
JMP FD	Check for failure processing, begin cycle calculations

4.1.6 Optional Program Modifications

The program modifications discussed below appear to be desirable for follow-on or more advanced testing. For the initial BB DDH tests, defined in this test plan, it is presumed that these changes will not be implemented. The purposes of the BB DDH testing can be accomplished with the current program modified as discussed in this section, and the procedures discussed in Sections 5 through 9.

4.1.6.1 SAR Gimbal Position Compensations

The SAR instrument error model differs from the strapdown model incorporated in the RSP in three ways:

1) The input axis mass unbalance drift error is a function of the SAR gimbal angle.

2) There is an error term (drift) M_{ep}, which is a function of acceleration along the output axis. This is also a function of gimbal angle.

3) The gyro input axis misalignment is a function of the gimbal angle.

The gimbal angle for each SAR is equal to the total (body fixed) input angle about that instrument input axis.

Compensation for the g sensitive drift terms would require:

1) Accumulation of total body fixed input angle (gimbal angle) for each SAR, Θ_I^G.

2) Modification of the gyro mass unbalance compensation matrix equation from the present form

$$\varepsilon i = (K_1 \underline{U}_{sA} + K_2 \underline{U}_{IA}) \cdot \underline{a}_s$$

to the form

$$\varepsilon i = (K_1 \underline{U}_{sA} + K_2 \underline{U}_{IA} + K_3 \underline{U}_{OA}) \cdot \underline{a}_s$$

where: $\underline{a}_s$ = sensed acceleration vector expressed in body fixed coordinates

K_1, K_2 = gyro mass unbalance compensation parameters

K_3 = output axis acceleration sensitivity

$\underline{U}_{sA}$ = the spin axis unit vector expressed in body fixed coordinates, a function of Θ_I^G.

$\underline{U}_{IA}$ = the input axis unit vector expressed in body fixed coordinates, a constant vector.

$\underline{U}_{OA}$ = the output axis unit vector expressed in body fixed coordinates, a function of Θ_I^G.

Compensations for input axis alignment as a function of gimbal angle would employ the same computed angles Θ_i^G in a modification of the equations for computing the angular rotation vector $\Delta\underline{\alpha}$, and the gyro test signals (V' matrix).

The equations and coding for these changes have not been developed but it is presumed that suitable forms can be found that can be incorporated in the present DDP-124 program.

4.1.6.2 Gyro Bias Estimator

If considerable time has elapsed since a complete gyro calibration, navigation performance can be improved by an update of the gyro drift estimate immediately prior to a navigation run. In BB DDH testing this estimate will be made after the fact from taped data processing using special procedures.

The RSP program could be modified to estimate the total drift (g sensitive plus fixed drift) about each axis, in the van, using an optically determined azimuth. The bias estimator would contain a filter for gyro and accelerometer quantization noise. The filter would be significantly different from the self-contained alignment filter presently in the RSP program. The self-contained alignment filter does not estimate gyro drift, and it is not designed to make use of optical measurements.

The size of the program required for bias estimation is comparable to the size of the present alignment routine.

4.1.6.3 Maximum Likelihood Failure Detection and Isolation

Recent developments in failure detection methods have shown optimal (maximum likelihood) filtering techniques to yield more efficient (reliable, accurate) system operation in the presence of degraded instrument performance (soft failures).

Using these techniques, and logic matched to the model and requirements of the MSFC SAR Dodecahedron navigation system, the RSP program could be modified for evaluation of more sophisticated and efficient failure isolation techniques.

A quick analysis of a set of maximum likelihood filtering equations indicates that this scheme can be incorporated in place of the present FDDC routine. The execution time for this scheme will be approximately equal to the FDDC routine. The area which needs to be investigated further is the quantization effects of the DDP-124 fixed point arithmetic on some of the filter variables.

4.1.6.4 Calibration Program Changes

If SAR gimbal angle compensations are employed, changes will be required in the RSP calibration routine and calibration procedures to measure output axis g sensitive drifts and SAR internal misalignment (two components) for each instrument.

The test control logic to select rate test, static test, compute, etc., could be revised to make use of the revised calibration sequence much more straightforward (Subparagraph 5.4.2.2).

4.2 SOFTWARE CHECKOUT

4.2.1 Copying and Tape Manipulator

Before starting checkout of the Redundant Sensor Program (RSP), the current symbolic tape and object tape should be copied and filed away. After a good copy has been made of the RSP tapes, the modifications (Subsection 4.1 above) to the program should be made and an assembly with a listing should be done. Once an error free assembly is obtained and all the modifications have been inserted in the proper place, an object tape should then be made. In order to do this, the FORTRAN Routines (RDWT and KFTR) will have to be put onto paper tape.

In order to create a new object tape, the system DAP II Assembly program must be loaded into memory and the following options selected:

Source input from tape

Object output on tape

Avoid rewind and End-of-file (EOF) on object tape

Bypass listing

The RSP tape should then be mounted on the input tape unit and a scratch tape on the output unit. The scratch tape will contain the object code. At the completion of the assembly, the RSP tape is removed, the system tape is mounted again and the FORTRAN compiler is selected. The following options should be selected:

Source input from paper tape

Object output on magnetic tape

Write an EOF and rewind object

Tape at completion

Bypass listing

At the completion of the compilation, the object tape will contain the complete RSP except for the system subroutines. In order to load the program into memory, put the system tape on one unit and the object tape on the other.

Select the loader with the following options:

Magnetic tape input

Select file one

Put loader map on the typewriter

At the completion of loading the program, it is possible that a system subroutine will be missing. This routine is ØØLD and it is a line printer dump routine. Missing this routine will not cause any problems as long as there is not a line printer attached to the DDP-124.

As part of the software checkout, it will doubtless be necessary to make octal dumps of memory. The RSP has an octal dump routine built into it. In order to use it, the flag TYFL needs to be set to 1, either with the input routine or the assembler. Also, the octal location of the dump needs to be noted.

To dump portions of memory onto the typewriter, set the location counter EQUAL to the octal location that corresponds to the symbolic location DUMP. The A register is set equal to the desired starting and stopping memory locations. Since the A register can only hold either octal digits, only the most significant address digits can be specified. For example, if an octal dump of memory from octal location 07010 through octal location 15672 is desired, first, transfer control to the DUMP routine, at this point the computer will halt. Next, the A register is set equal to the following:

07011570, and then the start button is pushed. Assuming that the dump flag has been set to dump onto the typewriter (TYFL equal to 1), the octal dump will then take place. At the conclusion of the dump, the routine returns to DUMP and halts. At this point, another dump can be initiated by setting the A register with another pair of addresses.

The above A register setting will actually dump from 07010 through 15700. Since the least significant octal number of the address cannot be specified, an address of 15700 was selected so that 15672 would be included in the dump.

4.2.2 Checkout of Alignment and Navigation Program

In order to do a detailed checkout of the program, a minimum of two runs are required. Both are nominal runs, one while in the alignment mode and the other in the normal mode. Both runs should be made on the original unmodified program and then repeated after all changes have been incorporated into the program. Also, the Driver/Navigation mode run should be made to checkout the consistency between the Driver and Navigation routines and to evaluate the larger quantization effects on the Navigation routine. Procedures for making these runs were described in Section 4.3.1, Reference 2. Expanded output word (OPW) lists for both runs and output for the first three seconds for the nominal run and the first six seconds for the alignment run of the present program are included in Appendix B.

4.2.3 Checkout of Calibration Program

Simulation runs shall be made as described in Paragraph 4.3.2, Reference 2.

4.2.4 Alignment Filter Simulation Runs

Prior to the alignment test phase (Section 6), simulated alignment runs, using the driver routine, shall be made to determine the alignment filter convergence time with the increased instrument quantization of the BB DDH.

Figures 4-1 and 4-2 show the results of previous simulation runs for various accelerometer quantization values, all smaller than the BB DDH quantization.

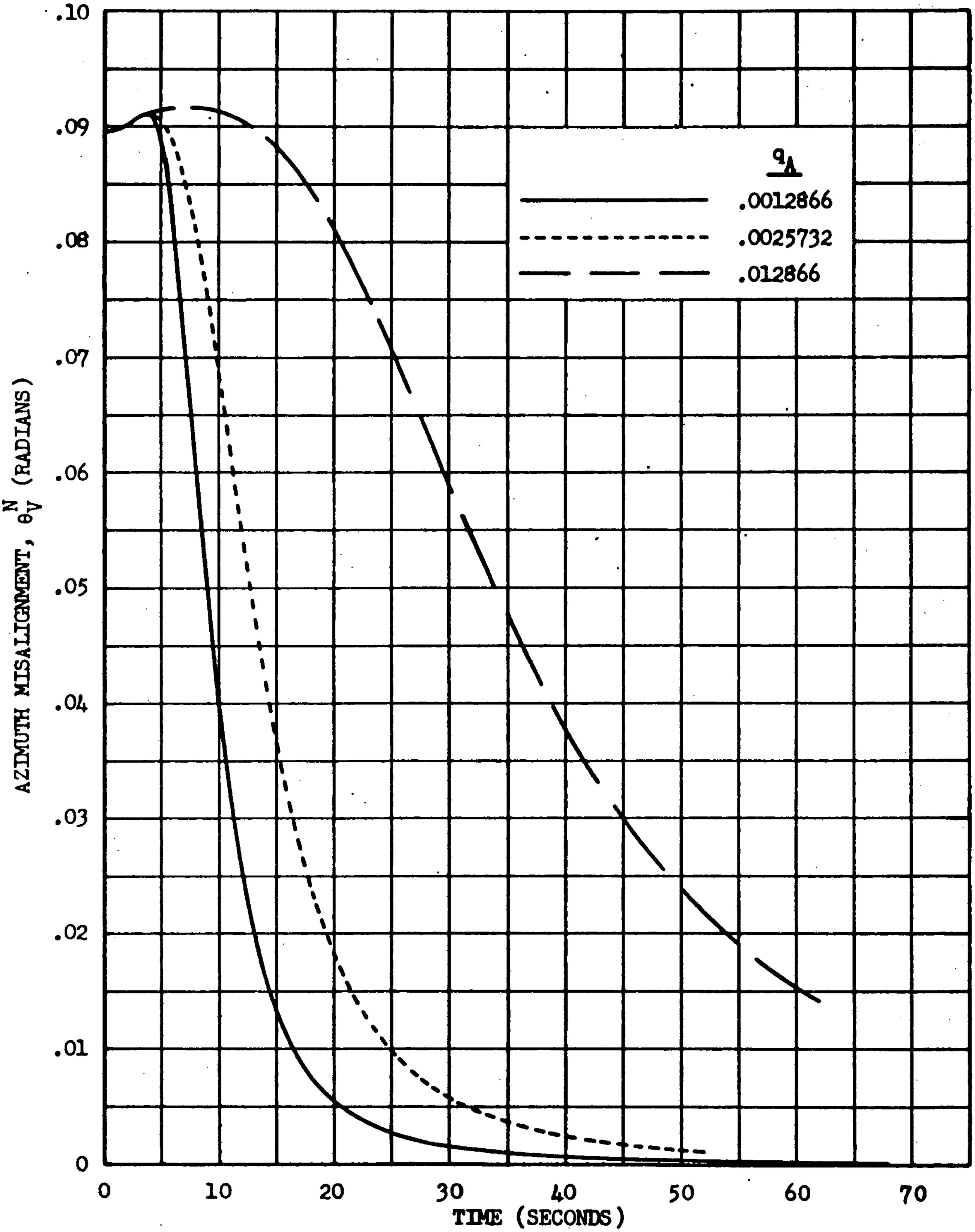


Figure 4-1. Azimuth Misalignment versus Time With No IMU Sway Motion

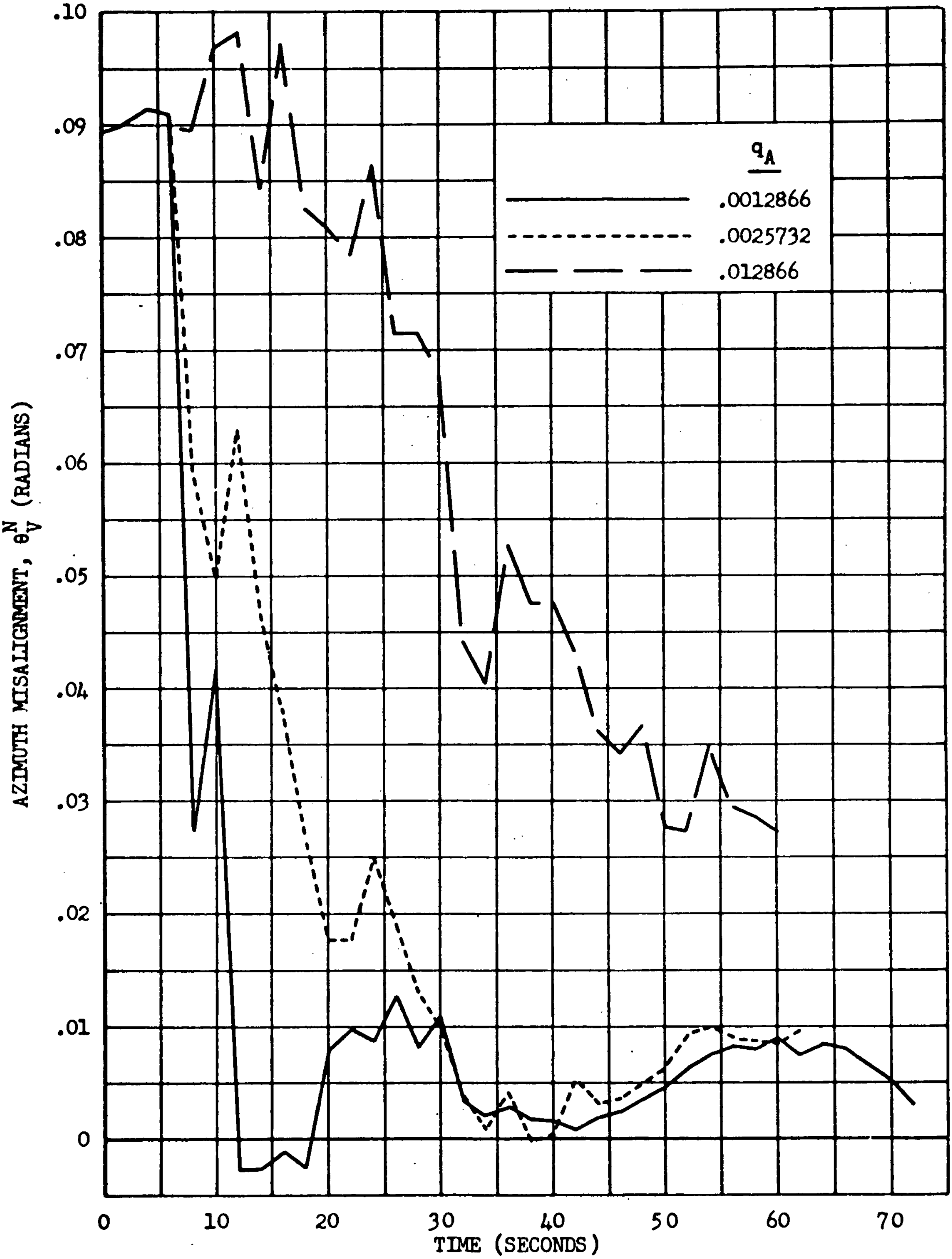


Figure 4-2. Azimuth Misalignment versus Time With High Amplitude, Low Frequency IMU Sway Motion

5. CALIBRATION TEST PHASE

5.1 TEST OBJECTIVES

The specific objectives of this test phase are to:

- Validate the modified RSP calibration routine, the calibration procedures, and all sign conventions, with the BB DDH.
- Obtain compensation coefficients for the RSP Align, Navigation, and Failure Detection Routines.
- Obtain performance data on:

  BB DDH parameter stability off-nominal performance; hysteresis, position sensitivity, etc.

  Calibration method accuracy

5.2 TEST REQUIREMENTS

The specific requirements of this test phase, to satisfy the test objectives, are:

- Validate compatibility of the calibration program, the BB DDH, and the test equipment interfaces.
- Validate the sign conventions, term by term, in the measurement and computation of all compensation parameters.
- Establish the accuracy of the calibration systems by 1) error analysis and 2) empirical confirmation.
- Determine compensation parameter coefficients for main program. Verify consistency with prior values for the same instruments.
- Obtain repeated and redundant data and analyze for:

  parameter repeatability

  measurement accuracy

  OA axis g sensitive drift

  position sensitivity

  hysteresis

  channel intercoupling

  temperature sensitivities

  other anomalies

- Determine the required recalibration interval for the balance of the test program.
- Develop operator skill in performing calibrations (a significant factor in inertial equipment testing).

5.3 TEST CONSTRAINTS

5.3.1 Software Status

The modified calibration program shall have been verified with test cases as defined in Section 4.

5.3.2 Hardware Status

The hardware shall have been verified to conform to the configuration of Section 2 and Reference 2, Section 3.

5.3.3 Inputs

The calibration test stimuli shall consist of earths rotation and gravitational acceleration in a six-position static test, and these plus rate table rotation in a three-position rate test.

5.4 TEST DESCRIPTION

Note: General procedures and test details will be contained in the Detailed Test Plan.

5.4.1 Calibration Test Setup

The nine required test positions, shown in Figure 5-1, are obtained by use of the adjustable test fixture described in Reference 2, Paragraph 2.3.3, and the two degrees of freedom of the GOERZ table (rotary axes and trunnion axis). Prior verification of the alignment (and stability) of the GOERZ rotary axis to the vertical in position 1, and to the horizontal and North in positions 4 through 9, is assumed. Trim of the BB DDH axes to the vertical and North after each remounting is accomplished with leveling adjustments on the BB DDH base and on the mounting fixture, while observing the alignment cube optically. Coarse alignment can be accomplished in [illegible] positions [illegible] ~~7~~, ~~6~~, and ~~7~~ using the [illegible] BB DDH bubble levels.
1, 4 and 9

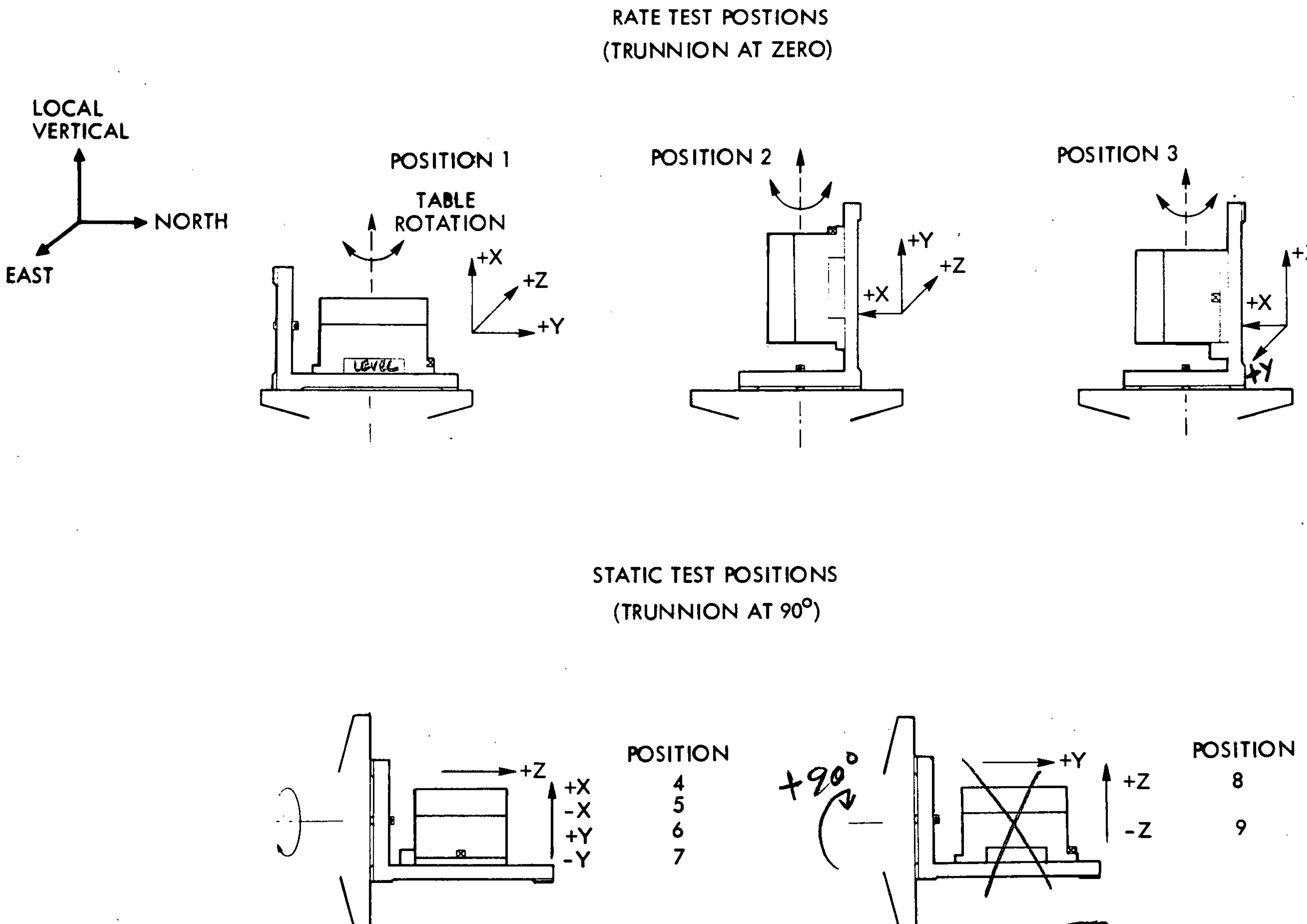
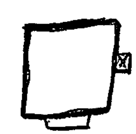


Figure 5-1. BB DDH Calibration Test Positions



The RSP requires sense switch settings to indicate which position the DDH is in and six angles Θ (i,j,k) indicating the azimuth of the horizontal axes in the static test positions (4 through 9) Table 5-I shows the set of azimuth angles Θ (i,j,k).

Table 5-II shows the correlation between sense switch settings, the CTFG (cal test flag), and test table positions in the STATIC TEST mode. Note that the measured sensor output subscripts 1 through 6 (Figure 3-9, Volume II, Reference 2) agree with the CTFG bit numbers.

Table 5-I. Azimuth of Assembly Axes

Equation Symbol	Nominal Azimuth Angle
Θ YXU	90 degrees (EAST)
Θ YXD	270
Θ ZYU	0
Θ ZYD	0
Θ XAU	90
Θ XZD	270

Table 5-II. Static Test Cal Flag (CTFG) Status vs Calibration Position

CTFG Bit & Sense SW (1)	Orientation	Position Number Figure 5-1
1	X up	4
2	X down	5
3	Y up	6
4	Y down	7
5	Z up	8
6	Z down	9

The gimbal of each SAR must be positioned prior to the static tests to locate the spin axis or output axes as shown in Table 5-III, and Figure 5-2, for compatibility with the calibration program. *Levels & alignment prisms are opposite that shown i.e. -Z side, not +Z side*

Table 5-III. SAR Gimbal Alignment in Calibration

SAR	DDH Reference Axis
X1, X2	Output axis along +Z
Y1, Y2	Spin axis along +X
Z1, Z2	Spin axis along -Y

5.4.2 Calibration Sequence

5.4.2.1 Nominal Sequence

Sensor alignment angles to the BB DDH reference axes are measured in the rate tests, positions 1, 2, and 3. These values are required in computation of static test results. The presently programmed sequence is rate tests, then static tests. The nominal sequence is:

		Position
1.	Rate Test Position	1
2.	Remount Fixture	
3.	Rate Test	2
4.	Remount DDH	
5.	Rate Test	3
6.	Compute Rate Test Results	
7.	Static Test	4
8.	Static Test	5
9.	Static Test	6
10.	Static Test	7
11.	Remount DDH	
12.	Static Test	8
13.	Static Test	9
14.	Compute Static Test Results	

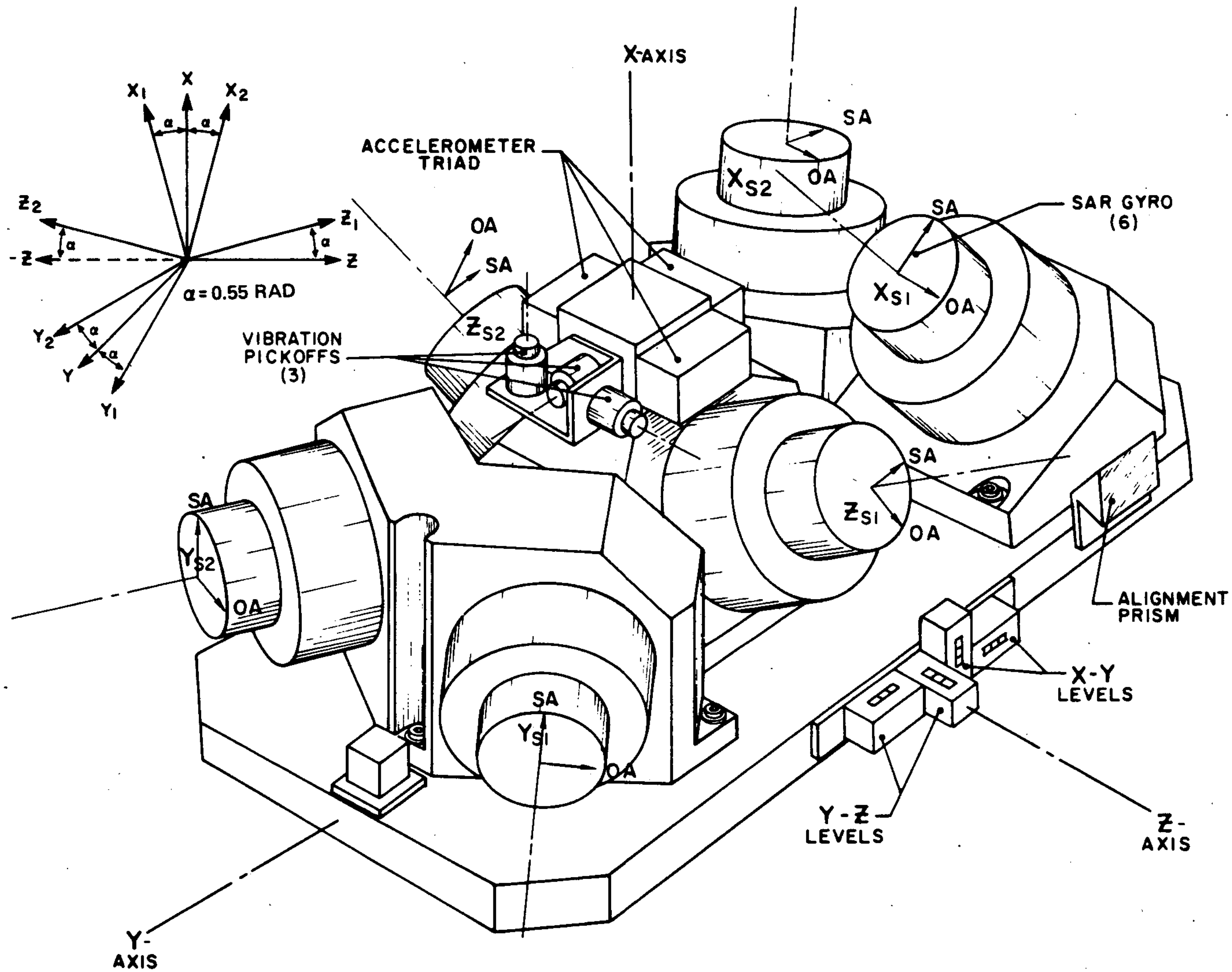


Figure 5-2. BB DDH Sensor Orientation

5.4.2.2 Abbreviated Sequence

A revised calibration sequence minimizing the number of remountings and the total test time, follows:

		Position
1.	Static Test	4
2.	Static Test	5
3.	Static Test	6
4.	Static Test	7
5.	Rate Test	3
6.	Remount DDH	
7.	Rate Test	2
8.	Static Test	8
9.	Static Test	9
10.	Remount Fixture	
11.	Rate Test	1
12.	Compute Rate Test Results	
13.	Compute Static Test Results	

This sequence requires considerable manipulation to exit from the static and rate test routines repeatedly; the calibration program logic could be changed to make this out of sequence operation much easier.

The static tests, and computation, can be performed separately by inputting (from paper tape) previously measured gyro scale factors and direction cosines.

5.4.3 Calibration Alignment Accuracy Requirements

An alignment error analysis has not been performed (see Paragraph 5.5.1), but preliminary estimates of the desired calibration test alignment errors are:

Alignment to North	3 arc minutes
Alignment to Table Rotation Axis	15 arc seconds
Table Rotational Axis to Vertical	10 arc seconds
Error in Axis Inversion	10 arc seconds

5.4.4 Calibration Test Output Data

The rate test and static test outputs are detailed in Tables 3-XV and 3-XVIII of Volume 1, Reference 3. They include:

Rate Test

Gyro Scale Factors	(6)
Gyro Direction Cosines	(6x3)
Gyro C Matrix	(15x6)

Static Test

Accelerometer Scale Factor	(6)
Accelerometer Biases-4 each	(24)
Accelerometer Direction Cosines	(6x3)
Accelerometer C Matrix	(bypassed)
Gyro Bias	(6)
Gyro Input Axis Mass Unbalance	(6)
Gyro Spin Axis Mass Unbalance	(6)

The four computed bias values for each accelerometer are for X, Y, and Z vertical, and the average of all three.

In addition, data is available for hand calculation of M_{ep} (drift proportional to acceleration along the gyro output axis), and (if data is taken in positions 10 and 11 (intermediate to 8 and 9)), redundant determinations of some of the above parameters are available.

5.4.5 Calibration Test Duration and Rates

The rate tests are to be performed at 10 degrees/second, clockwise and counterclockwise. The static test duration shall be 240 seconds. The static test interval could be increased to reduce the gyro drift measurement uncertainty (.02 degrees/hour) due to sensor pulse quantization (4.9 seconds), but this accuracy is sufficient for the breadboard testing.

A single calibration sequence, allowing time for shutdown, remounting, and temperature stabilization is estimated to require six hours.

5.4.6 Calibration Test Series

It is anticipated that two to three weeks will be required to obtain sufficient data to satisfy all of the test requirements listed in Subsection 5.2. Testing will consist of repetitions of all or part of the calibration sequence described above, and special investigations as required.

In the initial stages, raw data (pulse counts) will be read out frequently to confirm the calculations and particularly, to verify that the correct polarity is assigned to bias and g sensitive drift terms.

The test duration should be kept flexible and should last until the requirements are satisfied. Probably no less than 10 successful calibrations will be performed.

A flow diagram of the test and analysis sequence is shown in Figure 5-3.

5.5 CALIBRATION TEST ANALYSIS AND EVALUATION

5.5.1 Test Accuracy (Error) Analysis

A measurement error analysis, combining statistically the uncertainties in timing, position, counting, quantization noise, etc., should be performed to establish the expected accuracy in deriving the sensor compensation parameters.

5.5.2 Sign Validation

Sign errors in scale factors and direction cosines will be obvious. Each bias term (and particularly the g sensitive terms) should be examined for polarity by (1) comparing results with prior instrument test data, and (2) examination of the accumulated count data. Only if the error terms are very small will sign errors be undetectable.

5.5.3 Parameter Stability Evaluation Stability

Records of all calibration data should be maintained for analysis of repeatability and trending.

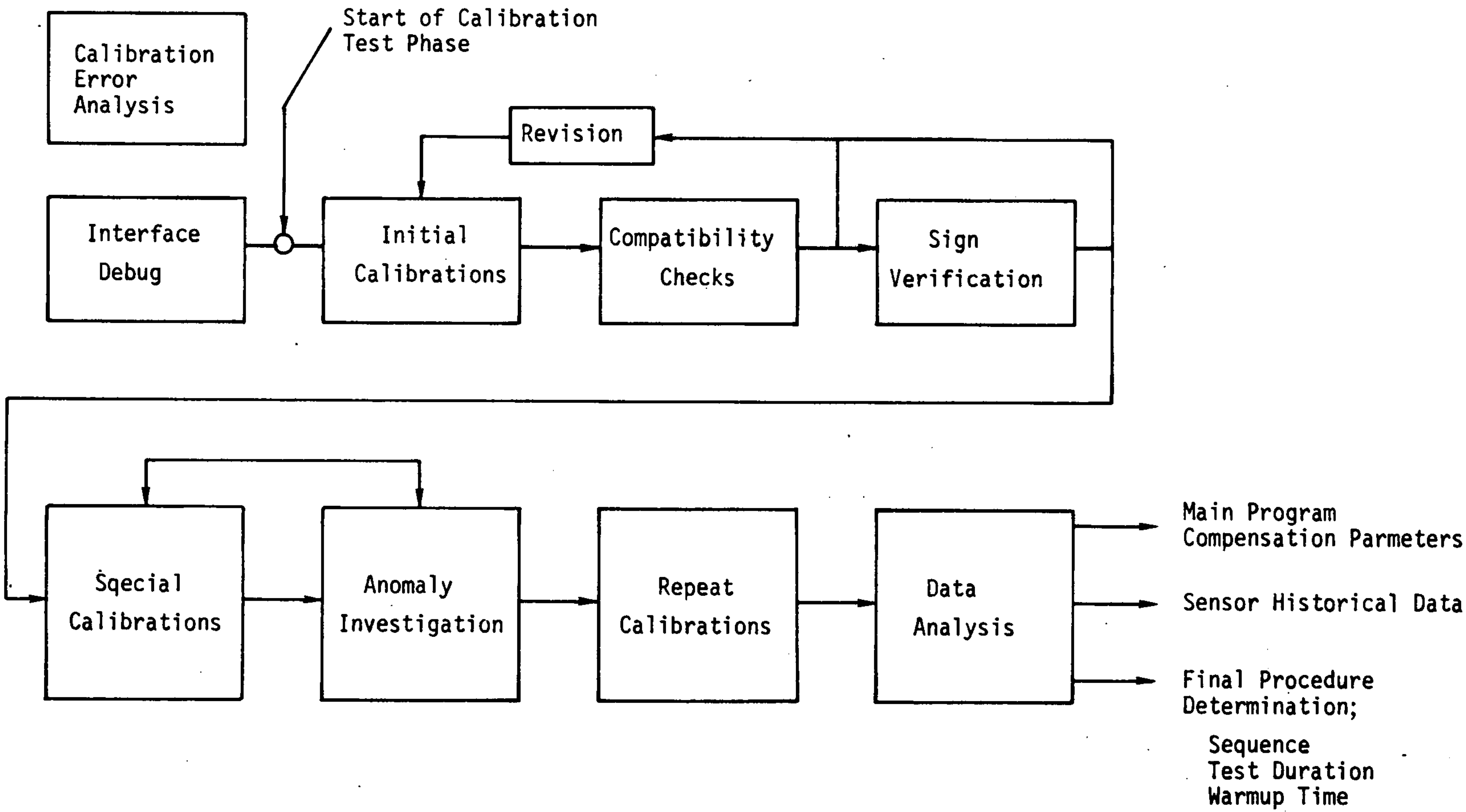


Figure 5-3. Calibration Test Flow Chart

Settling Time

The warmup time required for stable operation after each remounting should be determined empirically.

Anomalous Behavior

Redundant measurements exist for accelerometer bias, and can be derived for gyro bias. This data should be examined for consistency and evidence of position sensitivity or interchannel coupling.

If settling time, hysteresis or thermal lag type errors appear or are suspected in the calibration data, then special measurements should be implemented, varying the order or direction of rotation, thermal profile, or sampling interval. The three week estimate for the calibration test does not presume problems of this type.

6. ALIGNMENT TEST PHASE

6.1 TEST OBJECTIVES

In this test phase the accuracy and settling time of the self-alignment scheme is to be checked under static and (single axis) dynamic conditions.

Further alignment evaluation, with instrument failures is contained in Section 8, FDDC Tests. Alignment evaluation in the van environment is described in Section 9, Van Navigation Tests.

6.2 TEST REQUIREMENTS

The specific test requirements are:

- Validate compatibility of the alignment program and the BB DDH hardware.
- Evaluate alignment settling time with the large BB DDH sensor quantization values.
- Evaluate accuracy and settling time as a function of initial azimuth estimate error.
- Evaluate alignment accuracy and settling time with single axis oscillatory motion representative of stationary aircraft sway.

6.3 TEST CONSTRAINTS

6.3.1 Alignment Software Checkout

The modified driver program shall have been used to validate the alignment routine, and specifically to determine filter convergence (settling time) with the larger values of gyro and accelerometer quantization. These simulation results will dictate the nominal alignment time to be used in laboratory and van testing.

6.3.2 Valid Compensations

Alignment tests shall be preceded by a (recent) calibration. Update of gyro bias is particularly critical to alignment accuracy. If the calibration test analyses show that normal calibration procedures and delays will result in a total bias uncertainty during alignment in excess of .05 degree/hour, special procedures should be used to reduce the uncertainty.

6.3.3 SAR Gimbal (Head) Position

The standard SAR gimbal positions, to be established prior to each calibration run, are identified in Table 5-III.

6.4 TEST DESCRIPTION

6.4.1 Static Tests

6.4.1.1 Test Positions

The alignment tests shall be performed on the GOERZ Table. The standard test position shall be X vertical (up), Y North, and Z West (position 1 in Figure 5-1).

6.4.1.2 Test Duration

The nominal alignment interval will be established from the results of simulation (Driver) alignment runs.

6.4.1.3 Input Data

The site dependent data and actual BB DDH (and table) orientation angles shall be input, as indicated in Reference 3, Volume 1, Table 3-XIX, and as modified in Reference 2. Runs will be made with the initial azimuth estimate, $\hat{A}_Z$, in error by 0, 5, and 15 degrees. Compensation parameters shall be updated by the performance of a calibration.

6.4.1.4 Output Data

The principal real time output data will be the $\hat{A}$ matrix (body axis to VEN coordinates) which is updated by the alignment routine.

6.4.1.5 Reruns

Reruns, to be specified in the Detailed Test Plan, will be performed to evaluate:

- Repeatability
- Settling time versus initial azimuth error
- Accuracy versus bias compensation error (a polarity check)
- Accuracy versus alignment attitude

6.4.2 Dynamic Tests

With the GOERZ table trunnion at 90 degrees as in position 4, Figure 5-1, a low frequency horizontal axis oscillation, shall be used to simulate vehicle sway motion during alignment. The motion shall be terminated and the BB DDH returned to its starting position prior to the completion of alignment. The simulated sway motion shall be as specified in Table 6-I. BB DDH position, input and output data are as discussed under Static Tests.

Table 6-I. Simulated Sway Motion

Run	Sway-Amplitude 0-Peak	Frequency H_z
1	1.6 degrees	0.1
2	0.6 degrees	0.25
3	0.3 degrees	0.5

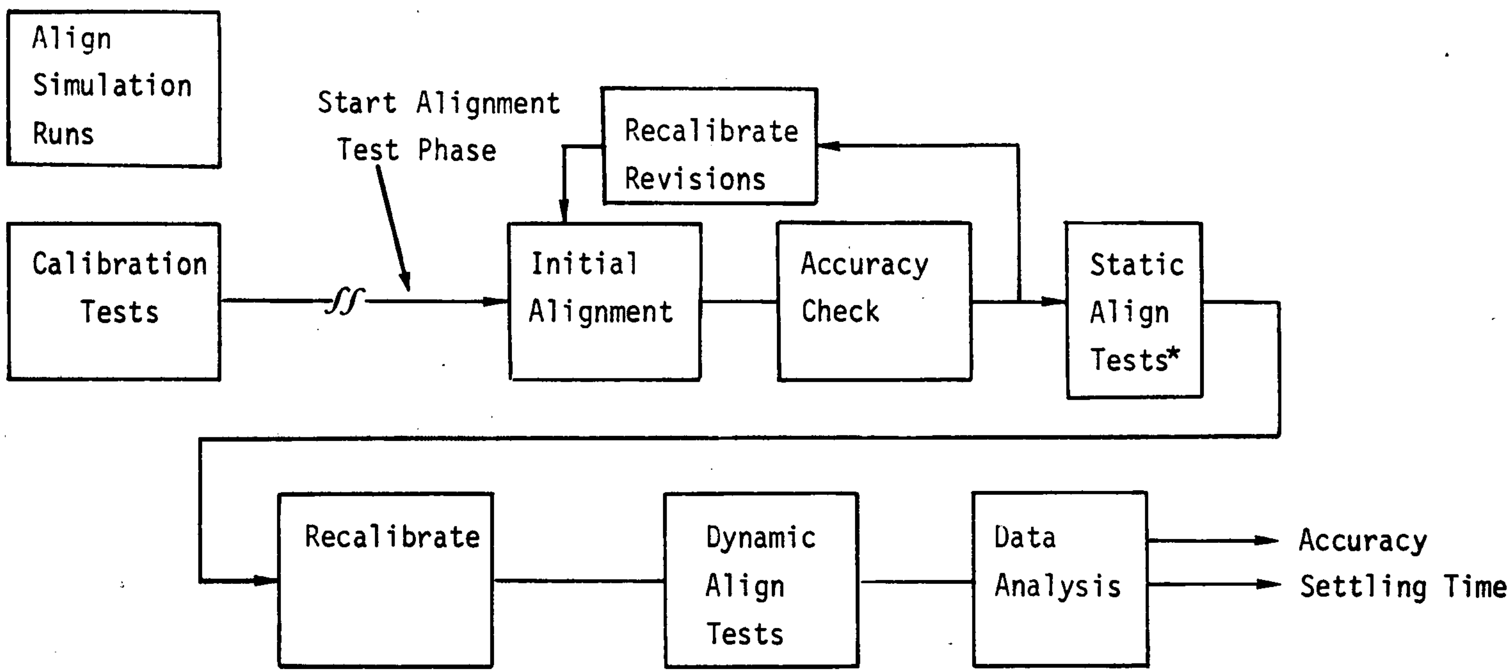
6.4.3 Test Sequence

Figure 6-1 is a flow diagram of alignment tests (without failures). It is anticipated that two weeks will be adequate time to evaluate the alignment routine and debug the procedures.

6.5 ALIGNMENT TEST ANALYSIS AND EVALUATION

The static and sway alignment data will be evaluated for consistency with simulation results under the various test conditions.

Alignment errors (in VEN coordinates) will be computed from the alignment attitude reference matrix A (body axes to VEN) and the known body to VEN attitude matrix K, by the cross product of the unit vectors:



* For various values of initial azimuth error.

Figure 6-1. Alignment Test Flow Chart

$$[\underline{u}_V \vdots \underline{u}_E \vdots \underline{u}_N] \triangleq K_{3x3}$$

$$[\hat{u}_V \vdots \hat{u}_E \vdots \hat{u}_N] \triangleq \hat{A}_{3x3}$$

$$\Theta_V = \hat{u}_E \cdot u_N$$

$$\Theta_E = \hat{\underline{u}}_N \cdot \underline{u}_V$$

$$\Theta_N = \hat{\underline{u}}_V \cdot \underline{u}_E$$

$$\Theta' = \begin{bmatrix} \Theta_V \\ \Theta_E \\ \Theta_N \end{bmatrix}$$

Similarly, alignment errors about body axes $\begin{bmatrix} \Theta_x \\ \Theta_y \\ \Theta_z \end{bmatrix}$ can be computed, for correlation with sensor errors.

The first of these computations is implemented in the existing program but a minor modification is required to make it usable in the real time align mode.

Convergence time can be monitored on the real time printout by noting when alignment is essentially complete, e.g., $\hat{A}$ matrix indicates alignment within .5 degrees of final value.

Alignment program output data will be recorded on magnetic tape. A useful external data evaluation tool would be a computer plotting routine for alignment errors.

7. NAVIGATION TEST PHASE

7.1 TEST OBJECTIVES

The objectives of this test phase are to validate the compatibility of the navigation program and the BB DDH hardware and to evaluate navigation performance without instrument failures.

7.2 TEST REQUIREMENTS

The specific test requirements are:

- Validate hardware/software compatibility and compensation adequacy.
- Evaluate navigation accuracy for static and single axis rotational tests, compare with expected values.
- Establish baseline data for FDDC tests.

7.3 TEST CONSTRAINTS

7.3.1 Sensor Compensation

The calibration interval required to maintain valid compensation coefficients during this test phase will be determined from analysis of the calibration test phase data analysis.

Navigation accuracy is particularly dependent upon accurate compensation for horizontal gyro drift. Total bias uncertainty (compensation error) in the navigation test orientation, should not exceed .05 degrees/hour per axis. Special calibration procedures should be used if stability between normal calibrations is inadequate.

7.3.2 Alignment

Navigation tests will normally be preceded by self-alignment. In special cases, designated in the Detail Test Plan, the alignment mode may be bypassed. The navigation program will then initialize to the input values which must accurately represent the BB DDH orientation.

7.3.3 Gimbal Positions

The standard SAR gimbal positions, to be established prior to each navigation run, are identified in Table 5-III.

7.4 TEST DESCRIPTION

Note: Effective test analysis and error identification requires that very simple navigation profiles be employed.

7.4.1 Test Position

The standard static test position shall be X up, Y north, and Z west (position 1 in Figure 5-1).

7.4.2 Test Duration

Eighty-four minutes are required for one cycle of errors related to gravity. Shofter term tests are useful for examination of certain effects. The two principal navigation intervals have, therefore, been chosen as 20 minutes and 90 minutes.

7.4.3 Input Data

The BB DDH (test table) orientation must be input, if different from the nominal program values. Nominal program values must have been established for the site geodetic parameters and other constants listed in Table 3-XXII, Volume I, Reference 3. Valid compensation parameters must be in the program from a recent calibration.

7.4.4 Test Method

For each navigation run (prior alignment assumed):

- insert initial attitude
- insert run duration
- initiate navigation
- introduce table motion (if required)
- observe velocity, position, or attitude errors
- tape selected word list
- terminate table motion
- terminate navigation
- printout final errors

7.4.5 Output Data

Real time output data is listed in Table 7-I.

Table 7-I. Navigation Test Real Time Output Data

Sense Switches 3	Sense Switches 4	Word 1	Word 2	Word 3	Parameter, VEN Coordinates
ON	OFF	Vert	East	North	Velocity Errors
OFF	ON	East	North	Altitude	Position Errors
ON	ON	Θ_V	Θ_E	Θ_N	

Taped output data selected from the major cycle output word list, Table 3-XXIII, Volume 1, Reference 3, will include:

- Latitude, longitude, VEN position, and velocity
- VEN position velocity and attitude errors
- Attitude matrix
- Test table rotation pulse accumulation

Minor cycle taped data will include:

- 6 gyro pulse accumulations
- 3 accelerometer pulse accumulations
- table angle
- discrete word

Taped data may be used in the tape input mode to make repeated navigation runs with the same sensor data.

7.4.6 Navigation Test Accuracy

Navigation accuracy will be limited by the instrument performance in the BB DDH thermal and mechanical environment. There is no closed loop temperature control. Lack of compensation for the SAR gimbal rotation will add some error but this will probably be masked by normal SAR instability.

Based on estimates in the error analysis section, the expected system navigation performance is in the 5 to 15 kilometers/hour range, depending on the bias update accuracy. These estimates are for simple navigation profiles, do not include dynamic errors, and presume constant ambient temperature.

7.4.7 Test Sequence

The test variables (to be specified in the Detailed Test Plan) will include:

Parameter	Nominal Value
Test Duration	90 minutes
Orientation(s)	Position 1
Rates	10 degrees/second
Alignment Method (Self or External)	Self
Altitude Damping Coefficients	$a_r = 1$ $a_v = 0$

Less time should be required for this test phase than for the alignment test phase as the main program/BB DDH interfaces will have been debugged.

7.4.7.1 Static Tests

Fixed attitude tests will be performed first to verify proper operation and compensation. Enough runs will be made to obtain statistics on repeatability.

7.4.7.2 Dynamic Tests

The basic dynamic test will involve a single rotation. This will demonstrate the errors to be encountered in van testing, but is still simple enough that reasonably accurate error predictions can be made.

The test table motion and rates will be specified in the Detailed Test Plan. A typical test profile will be:

1) Initiate navigation

2) Rotate 90 degrees at 10 degrees/second

3) Navigate in this fixed position for 90 minutes, terminate

A series of tests for different rotation angles will be made.

The second category of dynamic tests shall be static navigation with angular vibration about a horizontal axis, to evaluate system operation in the presence of simulated road motion. The vibratory amplitude shall be 1 degree/second at 5 Hz, or at the maximum frequency of which the GOERZ table is capable. When (if) actual measurement data on the van angular rotation environment becomes available, the laboratory test rates should be revised accordingly.

Figure 7-1 is a navigation test flow chart.

7.5 TEST ANALYSIS AND EVALUATION

Time profiles of the VEN navigation errors listed in Table 7-I will be the primary tool for navigation analysis. Computer plotting routines for these parameters (from tape) should be developed.

For each run, accuracy predictions should be made as in the error analysis, Appendix A, Section A4. By use of the error propagation equations and plots of Appendix A, identification of major sources of error in navigation runs should be possible.

Tapes from the runs make it possible to repeat the input data if a rerun is desired. This will be particularly useful in evaluating van data and FDDC performance.

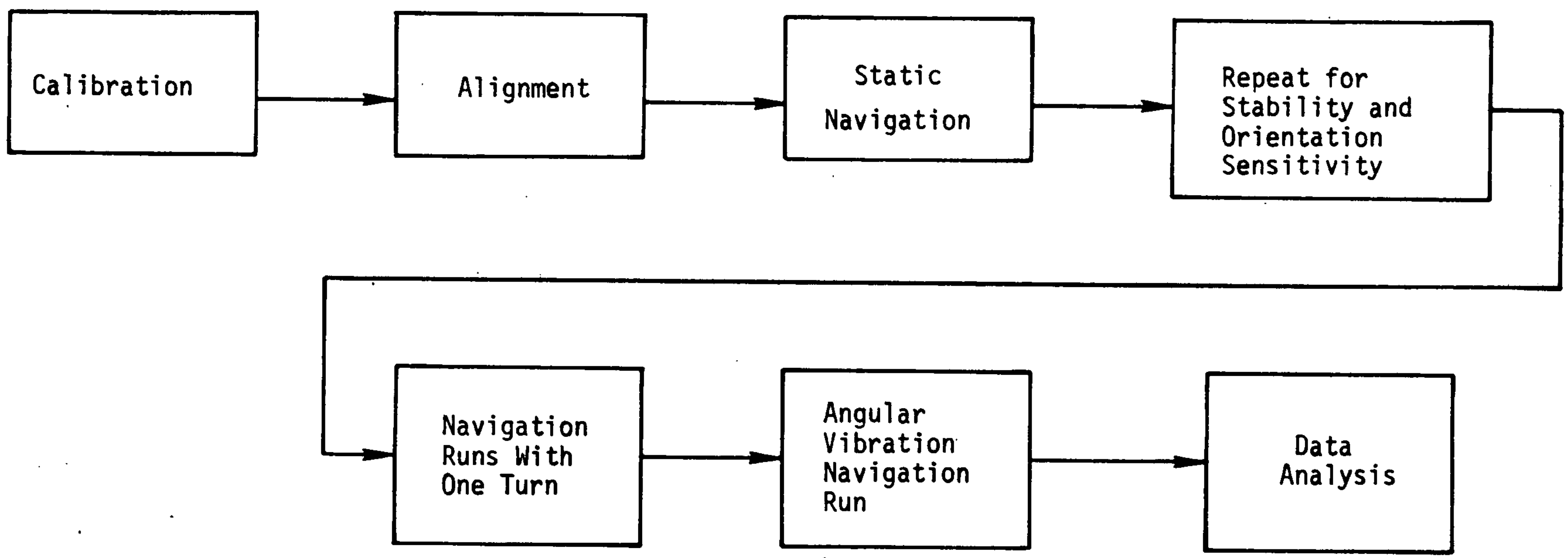


Figure 7-1. Navigation Test Flow Chart

8. FDDC TEST PHASE

8.1 TEST OBJECTIVES

The objective of this test phase is to evaluate performance of the FDDC logic with actual sensor outputs.

8.2 TEST REQUIREMENTS

The specific requirements of this test phase are:

- Establish empirically the minimum filter constants required to prevent false alarms.
- Evaluate failure detection time versus predicted time in the presence of noise.
- Evaluate navigation error with sensor failures.
- Evaluate navigation error with unfailed, but degraded sensors (below the detection threshold).
- Evaluate multiple instrument failures.
- Demonstrate internal monitor failure logic capability in isolating a third failure.
- Evaluate FDDC logic susceptibility to instrument shock.

8.3 TEST CONSTRAINTS

8.3.1 Sensor Compensations

Valid sensor compensation constants shall be in the program prior to each run. See Paragraph 7.3.1

8.3.2 Filter Constants

Initial values of the gyro filter constants ($Kf1_g$ and $Kf2_g$) have been established, based on the model of expected sensor performance and noise. See Paragraph 8.4.3.

8.4 TEST DESCRIPTION

8.4.1 General

In general, the FDDC tests will consist of repeats of the prior alignment and calibration tests (static and dynamic) with the introduction of one or more sensor failures. The effects of catastrophic failures are evaluated first, then the effects of degraded sensor performance.

8.4.2 Test Duration

Duration will be specified in the Detailed Test Plan. Some tests will be terminated as soon as a failure is detected.

8.4.3 Input Data

As for normal alignment and navigation tests, with the addition of the prefilter constants $Kf1_g$ (time constant) and $Kf2_g$ (gain or threshold). Normal values (from Appendix C) are:

$$Kf1_g = .99992$$

$$Kf2_g = 80 \text{ seconds/rad}$$

8.4.4 Test Method

For each real time test run:

- Set filter constants.
- Initiate alignment or calibration run as in Section 6 or 7.
- Introduce table motion (in dynamic tests).
- Tape record FDDC and navigation data.
- Introduce sensor failures.
- Observe the gyro failure state word (FSG) for time of failure.
- terminate motion, terminate run.

For taped data reruns:

- Set filter constants.
- Start tape.
- Initiate align or navigation run.
- Terminate run, terminate tape.

8.4.5 Output Data

The gyro failure state word can be monitored real time. The special output data which will be taped is listed in Table 3-XXVII, Volume 1, Reference 3.

Minor cycle (sensor 40 millisecond accumulations) will be tape recorded, so that repetitions of the same run can be made for filter parameter evaluation.

8.4.6 <u>Alignment Test Sequence</u>

The alignment tests will be repeated with catastrophic sensor errors, using output monitoring only. The sequence is:

- Static

  Repeat static alignment static test without failure as per Paragraph 6.4.1 (-Z axis east). Repeat static test with Z1 gyro failed at 30 seconds. (Disconnect gyro output from IFE). Repeat with Z1 failed at 30 seconds, Z2 failed at 35 seconds.

- Dynamic

  Repeat dynamic alignment test, run 2 of Paragraph 6.4.2, with no sensor failure. Repeat dynamic alignment one (easterly) gyro failed at 30 seconds. Repeat with one gyro failed at 30 seconds, a second failed at 35 seconds (both gyros will be those nearest the east pointing DDH axis). Test evaluation is discussed in Paragraph 8.4.8 below.

8.4.7 <u>Navigation Test Sequence</u>

The general sequence is:

- Verification of nominal filter threshold (gain) and time constants (no false alarms).
- Catastrophic failures (1 and 2 gyros failed) output comparison only:
  - Static navigation test
  - Dynamic navigation test (subparagraph 7.4.7.2).
- Catastrophic failures (1, 2, and 3 gyros failed), using internal monitoring only, static test only.
- Catastrophic failures (3 gyros failed), using both output and internal monitoring.
  - Static test only
  - Soft failures, using output comparison only, varying filter parameters and sensor error level
  - Shock susceptibility

The time of failure, the channels to be failed, and the magnitude (drift rate) of soft failures will be specified in the Detailed Test Plan. The BB DDH control panel is capable of introducing soft failures of .4 and 4 degrees per hour in any channel.

Figure 8-1 is a flow diagram of FDDC Testing.

8.4.7.1 Threshold Verification

The first test sequence will establish the margin of the nominal failure threshold level by lowering it until false alarms occur due to normal system performance. The nominal threshold will be raised if necessary to eliminate false alarms during static operation and during a turn of 90 degrees at 10 degrees/second.

8.4.7.2 Soft Failure Testing

This is the test phase where parametric experimentation will occur. The time required to detect a failure will be measured as a function of (1) instrument error and (2) failure threshold setting. The magnitude of navigation errors accumulated before detection of a failure will be measured. Probable values of instrument error and threshold setting are:

Instrument Error	0, .4, and 4.0 degrees/hour
Threshold Setting	.5, 1, and 2 times nominal

To provide repeatable data for evaluation of filter parameter changes, the input data shall be from tape, except for the first run.

8.4.7.3 Shock Susceptibility

The RSP failure detection logic locks up any of the 15 elements of the V vector (test signals) on the occurrence of a single check in excess of the threshold. There is some probability, therefore, of an accumulation of non-zero elements due to transients or shocks to the sensors, and ultimately switching out a sensor which had not permanently failed. A test shall be performed to examine the failure states (15 element V vector) with repeated shocks to the DDH, to evaluate the vulnerability of this FDDC logic scheme to noise and transients.

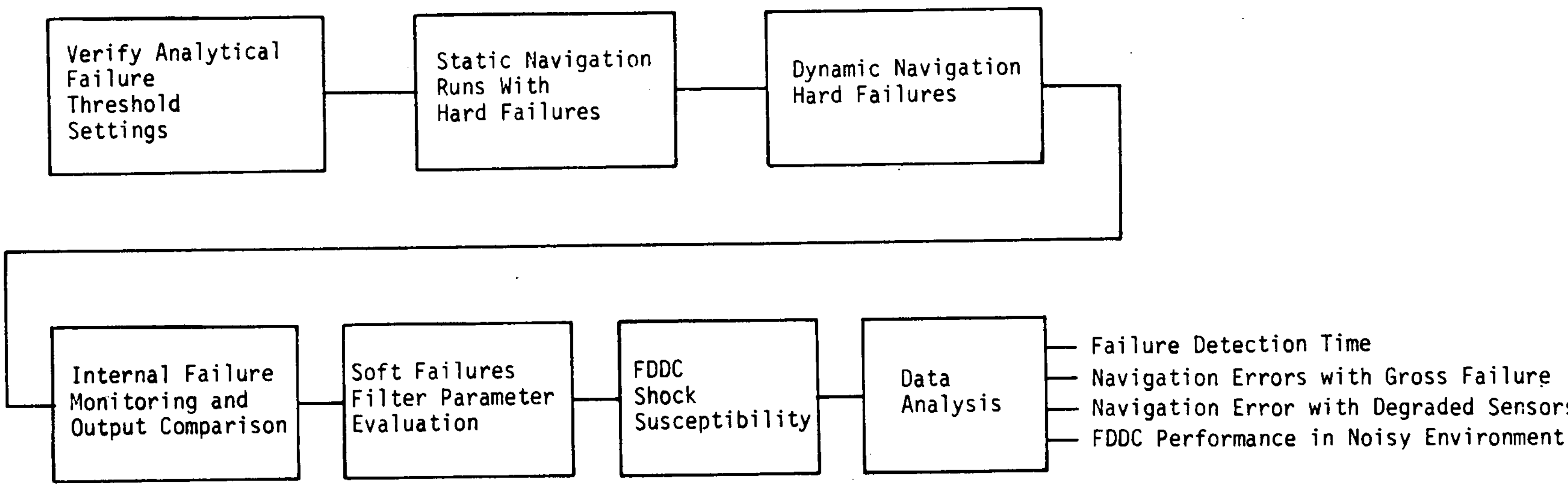


Figure 8-1. FDDC Test Flow Chart

8.5 TEST ANALYSIS AND EVALUATION

8.5.1 Selection of Nominal Filter Constants

Nominal initial values for the FDDC prefilter gain and time constant, derived in Appendix C, are:

$$Kf1_g = .99992$$

$$Kf2_g = 80$$

The corresponding time constant is 500 seconds. The detection threshold on the test signals, V_i', is .2 degrees/hour. Revisions to the nominal values may be based on the observed gyro (compensated) errors at the time of the FDDC test, using the method of Appendix C.

8.5.2 Navigation Accuracy With Sensor Failures

Predictions shall be made for navigation errors with 1, 2, or 3 sensors failed, using the methods of Appendix A, and results of tests shall be compared with predictions as discussed in Subsection 7.5. Anomalies shall be investigated by identifying the errors with the characteristic error functions given in Appendix A.

A special test and analysis that shall be performed to verify analytical results, is to increase sensor (gyro bias) errors to the maximum values that will not trigger the failure detection logic, then measure the navigation system errors in this degraded mode. The Failure Simulator routine will be used to inject these errors.

Failure detection times versus sensor errors and threshold levels should be evaluated against expected values using the methods of Appendix C.

From Appendix C it can be seen that for times long compared to the filter time constants (>30 minutes) the navigation error is limited (approximately) to the detection threshold rates, or .2 degrees/hour (approximately 22 Km/hour).

For times short relative to the time constant (<3 minutes) the navigation error is limited to the angle $\Theta = \tau/Kf \approx .028$ degrees or approximately 3.2 Km

9. VAN NAVIGATION TEST PHASE

9.1 TEST OBJECTIVES

The objective of this test phase is to demonstrate the system alignment, navigation, and failure detection/correction capability in a moving vehicle.

9.2 TEST REQUIREMENTS

The specific test requirements

- Establish an operating land navigation system with alignment and initialization techniques, in a mobile van.
- Obtain taped sensor data from typical vehicle test runs for repeated use in navigation program reruns in the laboratory computer.
- Evaluate self-alignment capability in the mobile van.
- Evaluate navigation capability in the van.
- Evaluate FDDC operation in the mobile van environment.

9.3 TEST CONSTRAINTS

9.3.1 Configuration

Real time navigation will not be done since the navigation computer will not be carried in the vehicle. Taped data will be used for input to the DDP-124.

Thermal and dynamic environment will not be tightly controlled. No privision now exists for measurement of angular vibration spectra in the van.

9.3.2 Sensor Compensation

Accuracy of the gyro compensation will probably be the limiting factor in navigation accuracy.

Prior to navigation runs if the BB DDH attitude is stationary and precisely known, by external alignment, total sensor biases can be extracted from the taped data, and used for compensation updates prior to making the taped navigation run. Procedures for determining gyro bias in this manner have not been established. The accuracy will be determined by the knowledge of BB DDH attitude at the time the reference data is recorded.

9.3.3 Gimbal Positions

Prior to vehicle alignment or navigation runs, the SAR gimbals will be set to standard reference positions (to be specified in the Detailed Test Plan).

On-board counters will be used to monitor and set the gimbal positions.

9.4 TEST CONFIGURATION

9.4.1 BB DDH Installation

The BB DDH will be mounted in the van with the X axis nominally vertical and the Y or Z axis parallel to the longitudinal (fore-aft) axis of the vehicle. Vibration isolators will not be used.

Provision will be made for optical access to the BB DDH alignment prism for external azimuth determination.

9.4.2 Van Instrumentation

The van instrumentation for measurement of auxilliary test data will consist of:

- tape recorder (14 track)
- gimbal angle readout (6)
- speedometer
- odometer
- thermometer
- vibration monitors

Considering the expected system accuracy (5 to 15 Km/hour) the vehicle odometer and speedometer will provide adequate reference data. A fifth wheel type precision odometer/speedometer is not required.

Accelerometers for linear vibration measurement are provided on the BB DDH. No provision currently exists for angular vibration measurement, although this data is potentially more useful (in the event of significant coning or angular rectification type errors).

9.5 TEST DESCRIPTION

9.5.1 Test Duration

The basic test interval shall be 90 minutes, to encompass one Schuler period. By establishing frequent checkpoints, the recorded data can also be used for evaluating navigation for shorter intervals. The necessary alignment interval will be determined from simulations (Paragraph 4.2.4).

9.5.2 Navigation Profiles

Error analysis will be enhanced by simple vehicle routines. e.g., right angle controlled-rate turns, and flat terrain. Some runs will be made with no maneuver constraints (other than maximum turning rates) but only gross evaluations will be possible. The course will be arranged to provide frequent surveyed checkpoint crossings.

Vehicle velocity will only be limited by the dynamic (vibration and shock) environment. Thirty mph (forty-eight Km/hour) will be the nominal cruising speed.

Vehicle turning rates will normally be limited to 90 degrees at 10 degrees/second to conform to laboratory tests and to avoid triggering the failure detection logic. In any case the turning rate must be less than 1 radian/second (the SAR limit) to avoid loss of inertial reference.

9.5.3 Bias Update

If an on-board bias update is desired prior to self-alignment or navigation, sensor data should be recorded for approximately 10 minutes in a stable precisely known attitude. A combination of BB DDH bubble levels, optical azimuth measurement, and vehicle maneuvering will be required to establish the attitude.

9.5.4 Navigation Coordinate System Alignment

Self-alignment will normally be used. However, by inputting the known BB DDH attitude as determined in Paragraph 9.5.3, if the self-alignment routine is bypassed, the navigation program will initialize (align) to the externally measured coordinates.

The externally measured alignment data will be used as a reference in evaluating self-alignment accuracy.

9.5.5 Input Data

When making computer navigation runs with taped data, the site dependent data shall be loaded as for laboratory navigation tests, Paragraph 7.4.3.

The input data normally representing BB DDH orientation on the GOERZ ($\underline{\phi}$, R^{E}_{IM}, R^{T}_{L}, and R^{L}_{I}) can be manipulated to represent the inertial orientation of the BB DDH, as measured optically (Paragraph 9.5.3) at the beginning of the navigation run. These inputs can be bypassed, as can external alignment and bias update, in van testing, with some decrease in accuracy.

9.5.6 Output Data

9.5.6.1 Van Data

Tape recorded data will include:

- 6 SAR outputs
- 3 accelerometer outputs
- time of day
- clock
- vibration instrumentation

Continuous tapes are desired. If necessary, tape changes can be made while stationary.

Manually recorded data will include:

- event-time log
- checkpoint ID, time

- velocity
- odometer heading
- gimbal angle readout
- temperature

9.5.6.2 Navigation Program Output Data

This will be the same as for laboratory tests, Paragraph 7.4.5.

9.5.7 Test Method

9.5.7.1 Van Navigation Recording Run

For each run:

- allow system temperature stabilization
- initialize SAR gimbal angles
- align vehicle with bubbles, external optics*
- record sensor data for alignment interval (to be determined) avoiding all motion
- realign gimbals
- drive prescribed course, recording sensor data
- induce sensor failures or errors at prescribed time**
- stop at checkpoints and for tape changes
- record environmental and event data
- return to starting point and realign*
- record static sensor data

9.5.7.2 Laboratory Computer Runs

- Initialize alignment program.
- Insert valid compensation coefficients.
- Run data tape (from van).

---

*Optional

**Failures (hard and soft) will generally be introduced at playback.

- From gyro sensor accumulations and known BB DDH alignment, compute gyro bias compensation error*.
- Update gyro bias compensation.
- Enter alignment routine.
- Record alignment data.
- Enter navigation routine.
- Induce failures as specified.
- Record data.
- Terminate run.

9.5.8 Test Sequence

9.5.8.1 General

After the van operation, procedures, recording equipment are debugged, relatively few navigation runs need be recorded. These runs (or portions of them) can be played back repeatedly, introducing any type of failure at will, to evaluate the align, navigation and FDDC routines. In these runs, the compensation parameters, alignment method, and filter parameters can be varied.

9.5.8.2 Static Runs

The first recorded van data will be in a stationary position with known BB DDH attitude for 90 minutes. This will be used to perform alignment and static navigation runs for comparison with laboratory test results.

9.5.8.3 Comparison Run for Laboratory Dynamic Tests

A run with a single 90-day rotation, preceded and followed by a static (alignment interval) should be performed for comparison with laboratory dynamic tests.

9.5.8.4 Maneuver Series

Tapes shall be recorded for a series of runs progressing in complexity from straight line constant speed to multiple turn variable

---

*Optional

speed. On some of these runs, internal monitor and gyro bias type failures should be introduced. Each run should be preceded and followed by a static data collection interval (for alignment or, if optically aligned, bias determination).

Ten or fewer 90-minute navigation tapes should provide all of the required data for navigation evaluation, except for the investigation of anomalies.

9.5.8.5 Laboratory Computer Runs (Alignment and Navigation)

The evaluation sequence shall be:

- static alignment
- static alignment with failures
- static navigation
- straight line navigation
- maneuvering navigation
- maneuvering navigation with failures
- FDDC tests

The FDDC tests will establish usable filter constants, and the failure detection effectiveness under field conditions.

Figure 9-1 is a flow diagram of the van test phase showing van tests and laboratory navigation runs.

If tape changes are necessary in navigation runs, a special procedure for suspending the navigation program in the absence of inputs will be required.

9.6 TEST ANALYSIS AND EVALUATION

9.6.1 Methods

The analysis methods will be the same as for laboratory alignment and navigation tests.

Accuracy, settling times, and failure detection thresholds and effectiveness will be re-evaluated in an operational environment.

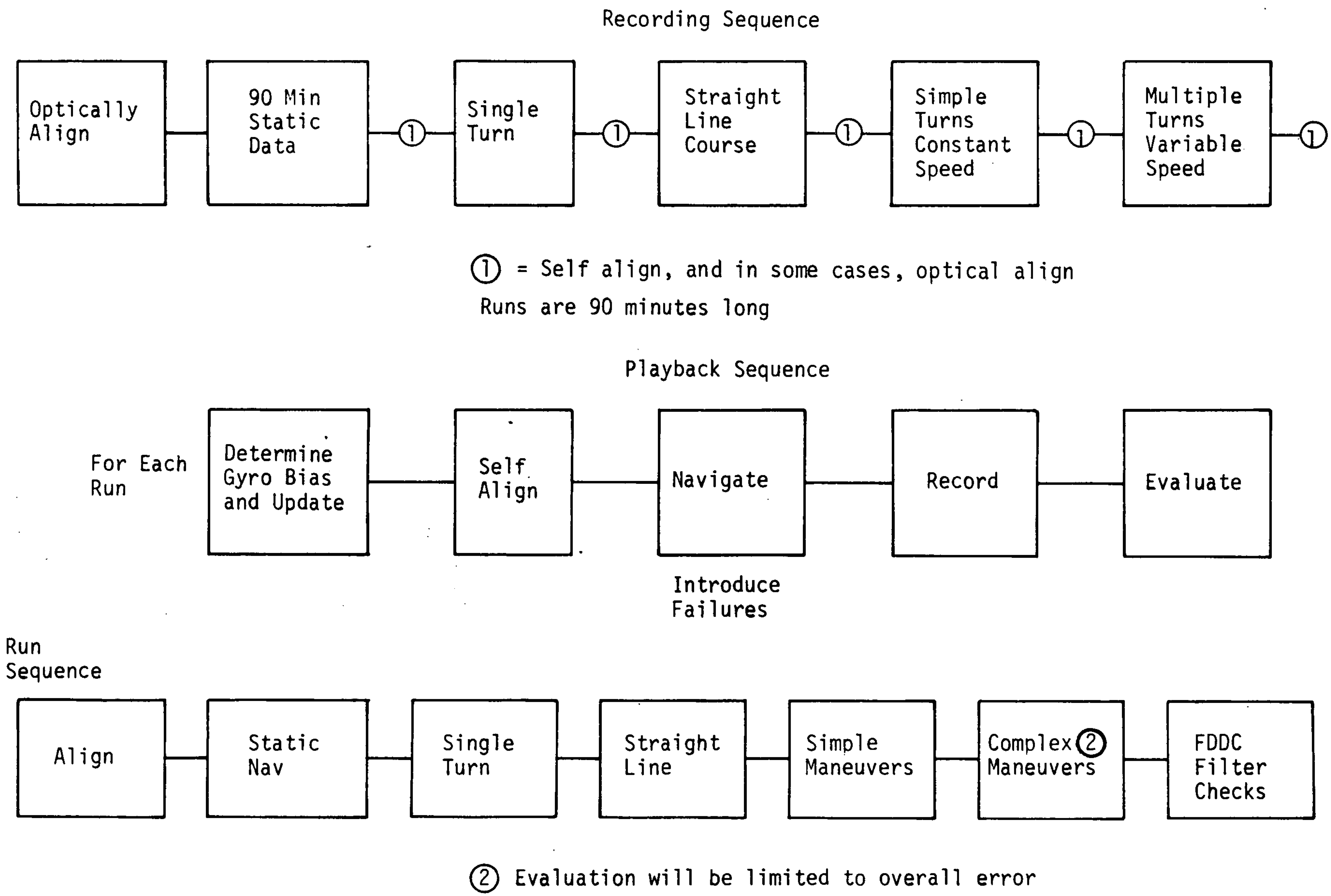


Figure 9-1. Van Test Phase Flow Chart

Computer plotting of navigation data will be especially valuable in attempting to relate maneuvers, navigation errors, and their sources.

9.6.2 Accuracy

The expected accuracy is less than in laboratory tests, probably in excess of 10 Km/hour for unconstrained maneuvers.

REFERENCES

1. "Single Axis Reference Strapdown Inertial Measuring Unit," MSFC NASA Technical Memorandum, NASA TM X-64586.

2. "Program Definition and Experiment Configuration Plan - Redundant Sensor IMU Evaluation Program" 23 September 1971, TRW Document No. 18313-6001-R0-00.

3. "Redundant Sensor Program Description," 30 March 1970.
   Volume I: Engineering Description
   Volume II: Programming Description
   Volume III: Program Listing

   "Interface Electronics Option Manual," Honeywell Document No. 130071943, March 1968.

   "Programmers Reference Manual, DDP-124 General Purpose Digital Computer," Honeywell Document No. 13007139A, April 1967.

4. "Redundant Sensor Program Description (ERSA/DDP-124), 27 May 1969.
   Volume I: Narrative Description
   Volume II: Appendices A, B, and C
   Volume III: Program Listing

APPENDIX A

ERROR ANALYSIS
REDUNDANT SENSOR STRAPDOWN IMU TESTS

A1.0 INTRODUCTION

The redundant sensor strapdown IMU tests will be performed in a test laboratory with a DDP-124 computer and in a mobile van. The test article will be the MFSC Breadboard Dodecahedron (BB DDH). The tests will be performed with the Redundant Sensor Computer Program (RSP) in the DDP-124.

The RSP was prepared for a series of tests which were to be performed at ERC with similar equipment. There are several differences between the equipment that will be used and the corresponding ERC equipment that will affect system performance.

- The BB DDH contains single axis references (SARs). The ERC test article contained single axis strapdown gyros. The SAR gyro is free to rotate with respect to the DDH structure, and the input axis mass unbalance drift and the cross coupling caused by SAR internal misalignments are both a function of SAR gimbal angle.
- The BB DDH contains three accelerometers with input axes nominally parallel to the DDH body axes. The ERC unit contained six accelerometers with input axes nominally parallel to the gyro input axes.
- The quantization of the BB DDH accelerometers and SARs is much greater than that of the sensors in the ERC unit. The quantization of the BB DDH accelerometers and SARs is 0.4 fps/pulse and 4.9439 arc seconds/pulse, respectively. (The quantization of the accelerometers will be reduced by a factor 5 to 10). The quantization of the corresponding sensors in the ERC unit is 0.001286 fps/pulse and 0.05 arc sec/pulse.

The laboratory tests will be performed with the GOERZ model 500 Test Table that was to be used in the ERC tests. In the forthcoming tests, the two axis fixture that was planned for the ERC tests will not be used. The test article will be mounted with a fixture such as that described in Section 2.3.3.2 of Reference 2. This type of fixture requires remounting during each calibration.

There are several characteristics of the RSP that will affect performance.

- The alignment filter in the RSP was designed for the quantization of the ERC unit. Convergence time will be increased by the larger quantization of the BB DDH.

- Gyro drift compensation in the RSP was designed for the ERC unit. There is no provision for applying gyro input axis mass unbalance compensation and SAR internal misalignment compensation as a function of SAR gimbal angle.

- There is no provision in the RSP for updating the gyro drift compensation during prenavigation alignment.

- There is no provision in the RSP for filtering the gyro and accelerometer outputs during calibration.

The performance capability of the inertial navigation system may be improved by suitable RSP changes. Such changes, however, are undesirable and will be held to a minimum. If changes can be avoided by test design, this approach is preferred.

In the interest of minimizing RSP changes, a navigation accuracy goal of 5 n.mi./hr will be assumed for the current test series. Calibration and alignment accuracies will be consistent with this goal. If greater accuracy should be required in later tests, additional RSP changes may be required at that time.

A brief analysis of the principal errors that will be generated in the current series of tests is presented in this appendix. The purposes of this analysis are as follows:

- Identify the RSP changes that will be required because of performance considerations.
- Provide test design guide lines. Define constraints that will avoid RSP changes. Provide a basis for determining calibration, alignment and mounting fixture accuracy requirements.
- Provide data that may be used in the evaluation of test results.

The analysis in this appendix is necessarily limited. This analysis should be extended by additional analyses as the program progresses.

A2.0 NAVIGATION ERROR ANALYSIS

A2.1 Introduction

Navigation errors will first be discussed for a nonredundant three axis strapdown IMU. It will be assumed that the IMU is oriented with two axes (Y and Z axes) level and the third axis (X axis) vertical. With the exception of the attitude drift that results from errors in the transformation of the earth rate vector from body coordinates to inertial coordinates, a nonrotating earth will be assumed.

Once the navigation errors of a nonredundant IMU have been developed, the discussion will be extended to a dodecahedron with no failures. The performance degradation associated with fault detection, diagnosis and correction (FDDC) will be discussed in Section A3.0.

A2.2 Gyro Drift, Level Gyros

If the orientation of the IMU is fixed, uncompensated constant drift of one of the level gyros will cause the following attitude error:

$$E_\phi = \frac{E_1}{\omega_0} \sin \omega_0 t \qquad \text{(A-1)}$$

where: E_ϕ = attitude error, rad

E_1 = level gyro drift, rad/sec

$$\omega_o = \sqrt{\frac{g}{R}} \tag{A-2}$$

where: g = gravitational acceleration, ft/sec^2

R = earth radius, ft

$$\omega_o = \sqrt{\frac{32.2}{2.09\text{x}10^7}} = 1.24 \text{ x } 10^{-3} \text{ rad/sec}$$

$$T_o = \frac{2\pi}{\omega_o}$$

$$= \frac{2\pi}{1.24\text{x}10^{-3}} = 5.06 \text{ x } 10^3 \text{ sec}$$

$$= \frac{5.06\text{x}10^3}{60} = 84 \text{ min} \tag{A-3}$$

The attitude error E_ϕ thus is periodic with period T_o(84 min).

With the orientation of the IMU fixed, uncompensated drift of one of the level gyros will result in the following velocity and position errors:

$$E_v = E_1 R\left[1 - \cos \omega_o t\right] \tag{A-4}$$

$$E_p = E_1 R\left[t - \frac{1}{\omega_o} \sin \omega_o t\right] \tag{A-5}$$

where: E_v = velocity error, ft/sec

E_p = position error, ft

Y gyro drift will result in velocity and position errors in the Z axis. Conversely, Z gyro drift will result in velocity and position errors in the Y axis.

The velocity and position errors both contain periodic terms with period T_o. The velocity error also contains the constant term E_1R. The corresponding position error term is the linear term E_1Rt.

The error sensitivity of the constant term in Equation (A-4) expressed in units of (n.mi./hr)/(degree/hour) is as follows:

$$\frac{E_v'}{E_1'} = R$$

$$= \frac{(2.09\times10^{7})(4.85\times10^{-6})(3{,}600)}{6{,}000}$$

$$= 60.8 \ \frac{\text{n.mi./hr}}{\text{deg/hr}} \qquad \text{(A-6)}$$

where: E_v' = velocity error, n.mi./hr

E_1' = level gyro drift, degree/hour

The attitude, velocity and position errors (assuming no IMU orientation change) for a gyro drift of .01 degree/hour are plotted in Figures 3-1, 3-2 and 3-3 of the main body of this document.

A2.3 Azimuth Alignment Error

In the RSP, the direction cosine matrix relating inertial coordinates to body coordinates (A matrix) is initialized by means of the alignment subroutine. An error in the initialization process can be viewed as an initial alignment error. Such an error may cause an error in the transformation of the earth rate vector from body coordinates to inertial coordinates and thus cause attitude drift.

The attitude drift in body coordinates caused by an initial azimuth alignment error is as follows:

$$E_r = E_2 \, \Omega \cos \lambda \sin \psi \qquad \text{(A-7)}$$

where: E_r = attitude drift, rad/sec

E_2 = azimuth alignment error, rad

Ω = earth's rotation rate, rad/sec

λ = test latitude, deg

ψ = angular displacement of level body axis from north, deg.

The earth rate transformation error will cause attitude, velocity and position errors. These may be computed with Equations A1, A4 and A5.

The error sensitivity of the constant velocity error term (Equation A4) expressed in units of (n.mi./hr)/arc seconds is as follows: (λ is assumed to be 35 deg and ψ is assumed to be 90 deg)

$$E_V' = R\ \Omega \cos 35° \sin 90°$$

$$E_2' = \frac{(2.09\text{x}10^7)(4.85\text{x}10^{-6})(3{,}600)(15)(4.85\text{x}10^{-6})(.819)}{6{,}000}$$

$$= .0036\ \frac{\text{n.mi./hr}}{\text{arc sec}}$$

where: E_2' = azimuth alignment error, arc second.

In the RSP, the reference coordinate system is aligned in azimuth by gyrocompassing. Uncompensated drift of one or both of the level gyros will thus cause an azimuth alignment error. This alignment error, in turn, will cause an earth rate transformation error that will at least partially offset the uncompensated gyro drift. Once navigation is initiated, this error offset will continue until the orientation of the IMU is changed. A change in IMU orientation will cause the earth rate transformation error transformed to body coordinates to change, and thus cause the total drift to increase.

This error mechanism will be illustrated by a special case. Assume that during alignment and during the initial portion of the navigation period, the IMU is positioned with Z axis north and Y axis east as shown in Figure A-1. At the termination of alignment, the drift in each body axis will be:

$$E = E_1 - E_c$$

$$= E_1 - E_2\ \Omega \cos \lambda \sin \psi$$

where: E = total drift, rad/sec

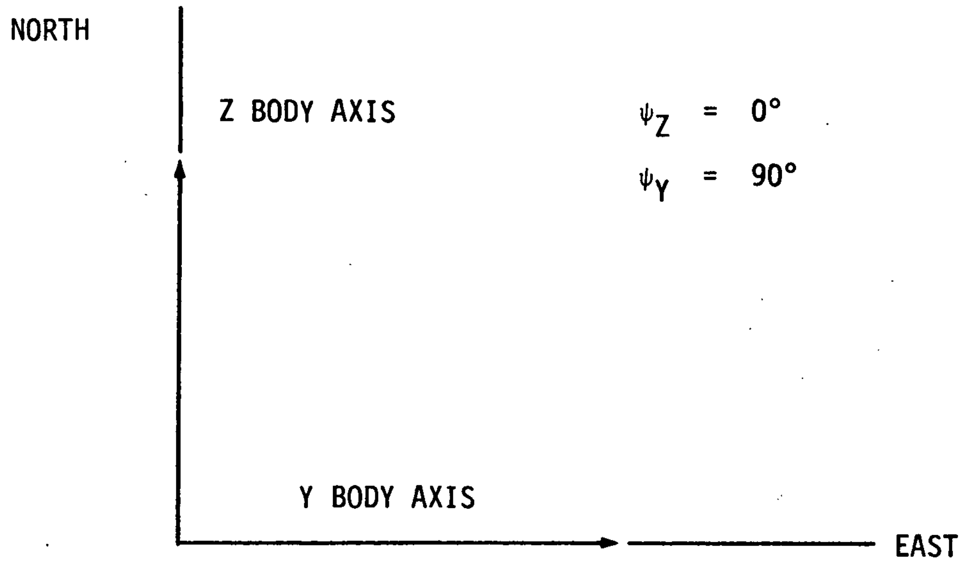


Figure A-1

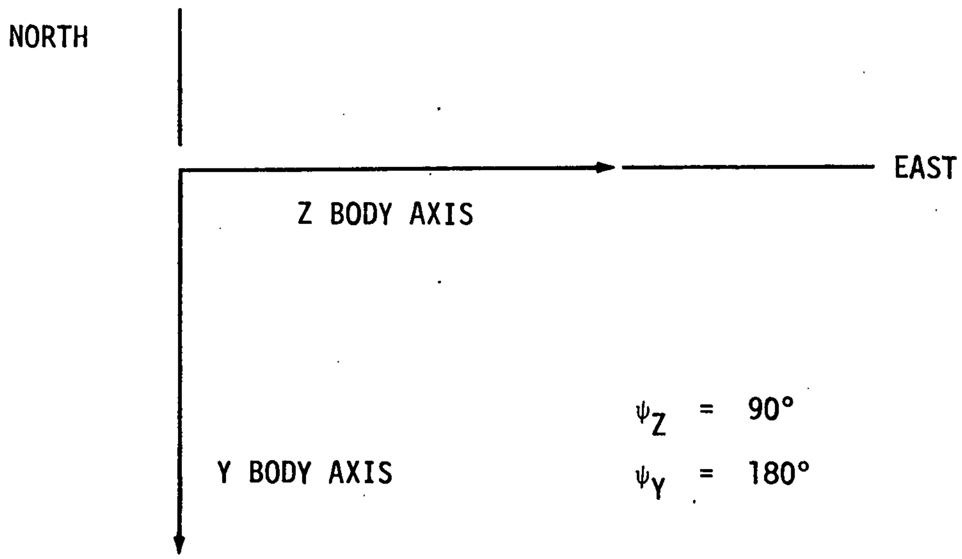


Figure A-2

In the Y axis (ψ = 90 deg),

$$E_y = E_{1y} - E_2 \, \Omega \cos \lambda$$

The alignment process will result in a misalignment E_2 such that

$$E_2 \, \Omega \cos \lambda = E_{1y}$$

and

$$E_y = 0$$

In the Z axis (ψ = 0 deg),

$$E_z = E_{1z}$$

Assume that during the navigation period, the IMU is rotated 90 deg about the vertical axis as shown in Figure A-2. In the Y body axis after rotation (ψ = 180 deg),

$$E_y = E_{1y}$$

In the Z axis (ψ = 90 deg),

$$E_z = E_{1z} - E_{1y}$$

The 90 deg rotation has thus caused the magnitude of both E_y and E_z to increase. (E_{1y} and E_{1z} are assumed to be statistically independent).

In addition to the foregoing errors, an azimuth alignment error will cause a position error that is a function of distance from the starting point.

$$E_p = E_2 d \tag{A-9}$$

where: d = distance from starting point, ft.

This is the total error (the rss of the Y and Z axis errors). This error will be much smaller than the position error resulting from the earth rate transformation error.

A2.4 Gyro Drift, Vertical Gyro

Uncompensated constant drift of the vertical gyro will cause an earth rate transformation error that changes linearly with time. The error component in each level body axis will be as follows:

$$E_r = E_3 t \, \Omega \cos \lambda \sin \psi \qquad \text{(A-10)}$$

where: E_3 = vertical gyro drift, rad/sec.

If the orientation of the IMU is not changed, the attitude, velocity and position errors that result from the foregoing error are as follows:

$$E_\phi = \frac{E_3 \, \Omega \cos \lambda \sin \psi}{\omega_o^2} \left[1 - \cos \omega_o t \right] \qquad \text{(A-11)}$$

$$E_v = E_3 \, R \, \Omega \cos \lambda \sin \psi \left[t - \frac{1}{\omega_o} \sin \omega_o t \right] \qquad \text{(A-12)}$$

$$E_p = E_3 \, R \, \Omega \cos \lambda \sin \psi \left[\frac{t^2}{2} - \frac{1}{\omega_o^2} (1 - \cos \omega_o t) \right] \qquad \text{(A-13)}$$

The foregoing equations apply to both the Y and Z axes. For ψ defined for the Y axis, E applies to the Y axis, and E_v and E_p apply to the Z axis. Conversely, for ψ defined for the Z axis, E_ϕ applies to the Z axis, and E_v and E_p apply to the Y axis.

The attitude, velocity and position errors all contain periodic terms with period T_o. In addition, the attitude and position errors contain constant terms, the velocity error a linear term, and the position error a second order term.

The error sensitivity of the linear term in Equation A-12 evaluated at t = 45 min and expressed in units of (n.mi./hr)/(degrees/hour)is as follows: (λ is assumed to be 34 deg and ψ is assumed to be 90 deg)

$$\left.\frac{E_V'}{E_3'}\right|_{t = 45 \text{ min}} = R\ \Omega\ (\cos 35°)(\sin 90°)(45)(60)$$

$$= \frac{(2.09\text{x}10^{7})(4.85\text{x}10^{-6})(3,600)(15)(4.85\text{x}10^{-6})(.819)(45)(60)}{6,000}$$

$$= 9.7\ \frac{\text{n.mi./hr}}{\text{deg/hr}} \qquad \text{(A-14)}$$

where: E_3' = vertical gyro drift, degree/hour

The attitude, velocity and position errors (assuming no IMU orientation changes) for a vertical gyro drift of .01 degrees/hour are plotted in Figures 3-4, 3-5 and 3-6.

In addition to the foregoing errors, vertical gyro drift will cause a position error that is a function of velocity history during the navigation period. For the special case of IMU velocity (magnitude and direction) constant, the position error is as follows:

$$E_p = \frac{E_3\ dt}{2} \qquad \text{(A-15)}$$

This is the total error (the rss of the Y and Z axis errors). This error will be much smaller than the position error resulting from the earth rate transformation error.

A2.5 Gyro Misalignment

If the IMU is rotated about the X axis, a misalignment of the Y gyro in the XY plane will cause an attitude error in the Y gyro axis. The same will be true for any other axis pair. Thus,

$$E_\phi = E_4\theta \qquad \text{(A-16)}$$

where: E_4 = gyro misalignment, rad

θ = IMU angular displacement, rad.

Rotation about the vertical axis will cause a level error in each of the two level gyro axes. The attitude, velocity and position errors resulting from a level error are discussed in Section A2.7.

Rotation about one of the level gyro axes will cause both an azimuth error and a level error. The attitude, velocity and position errors resulting from an azimuth error are discussed in Section A2.3.

The error sensitivity for rotation about the vertical axis expressed in units of n.mi./arc seconds is as follows: (A rotation of 360 deg is assumed.)

$$\frac{E_p'}{E_4'} = R\theta$$

$$= \frac{(2.09\text{x}10^7)(4.85\text{x}10^{-6})(2\pi)}{6{,}000}$$

$$= .106 \ \frac{\text{n.mi.}}{\text{arc sec}} \qquad \text{(A-17)}$$

where: E_p' = position error, n.mi.

E_4' = level gyro misalignment, arc sec

In addition to the foregoing errors, misalignment of the gyros with respect to the accelerometers and misalignment of the level gyros with respect to each other will cause the gyros to sense an erroneous earth rate component. This is a consequence of aligning by accelerometer leveling and gyrocompassing. The attitude, velocity and position errors that result from these earth rate sensing errors may be computed with Equations A-1, A-4, A-5, A-11, A-12, and A-13.

A2.6 Accelerometer Bias, Level Accelerometer

If the orientation of the IMU is not changed, an uncompensated level accelerometer bias will cause the following attitude, velocity and position errors:

$$E_\phi = \frac{E_5}{g}\left[1 - \cos \omega_o t\right] \qquad \text{(A-18)}$$

$$E_v = \frac{E_5}{\omega_o} \sin \omega_o t \qquad \text{(A-19)}$$

$$E_p = \frac{E_5}{\omega_o^{\ 2}}\left[1 - \cos \omega_o t\right] \qquad \text{(A-20)}$$

where: E_5 = level accelerometer bias, g

A Y accelerometer bias error will result in a Z axis attitude error and Y axis velocity and position errors. Conversely, a Z accelerometer bias error will result in a Y axis attitude error and Z axis velocity and position errors.

The attitude, velocity and position errors all contain periodic terms with period T_0. In addition, the attitude and position errors contain constant terms.

The error sensitivity of the constant term in Equation A-20 expressed in units of n.mi./μg is as follows:

$$\frac{E_p'}{E_5'} = \frac{1}{\omega_0^2} = \frac{R}{g}$$

$$= \frac{(2.09 \times 10^7)(10^{-6})}{6{,}000}$$

$$= .0035 \frac{\text{n.mi.}}{\mu g} \qquad \text{(A-21)}$$

where: E_5' = level accelerometer bias, μg

The attitude, velocity and position errors for an accelerometer bias error of 10 μg are plotted in Figures 3-7, 3-8 and 3-9.

A2.7 Level Alignment Error

An error in level alignment will cause a gravity computation error. The equivalent level accelerometer bias error is as follows:

$$E_b = E_6 g \qquad \text{(A-22)}$$

where: E_b = level accelerometer bias error, g

E_6 = level alignment error, rad.

A Y axis alignment error will result in a Z accelerometer bias error. Conversely, a Z axis alignment error will result in a Y accelerometer bias error.

The accelerometer bias errors will cause attitude, velocity and position errors. These errors may be computed with Equations A-18, A-19 and A-20.

The error sensitivity of the constant position error term (Equation A-20) expressed in units of n.mi./arc seconds is as follows:

$$\frac{E_p'}{E_6'} = R$$

$$\frac{E_p'}{E_6'} = \frac{(2.09 \times 10^7)(4.85 \times 10^{-6})}{6{,}000}$$

$$= .017 \ \frac{\text{n.mi.}}{\text{arc sec}} \qquad \text{(A-23)}$$

where: E_6' = level alignment error, sec

In the RSP, the reference coordinate system is level aligned by means of the accelerometers. Thus, a level accelerometer bias error will cause a level alignment error. This alignment error, in turn, will cause a gravity computation error that will offset the accelerometer bias error. Once navigation is initiated, this error offset will continue until the orientation of the IMU is changed. IMU rotation about a level axis will cause one of the two offsetting errors to change and thus cause the total error to increase. This offsetting error relationship is similar to that for azimuth alignment discussed in Section A2.3.

In addition to the foregoing errors, a level alignment error will cause an earth rate transformation error. The error component in each of the level reference axes is as follows:

$$E_r = E_6 \ \Omega \sin \lambda \qquad \text{(A-24)}$$

The corresponding errors in the vertical axis are as follows:

$$E_r = E_6 \ \Omega \cos \lambda \sin \psi \qquad \text{(A-25)}$$

The earth rate transformation error will cause attitude, velocity, and position errors. The errors caused by the level axis components may be computed with Equations A-1, A-4, and A-5. The errors caused by the vertical axis components may be computed with Equations A-11, A-12, and A-13.

The error sensitivity of the constant velocity error term in Equation A-4 expressed in units of (n.mi./hr)/arc seconds is as follows: (λ is assumed to be 35 deg).

$$\frac{E_v'}{E_6'} = R\,\Omega \sin 35°$$

$$= \frac{(2.09\text{x}10^{7})(4.85\text{x}10^{-6})(3{,}600)(15)(4.85\text{x}10^{-6})(.574)}{6{,}000}$$

$$= .0025\ \frac{\text{n.mi./hr}}{\text{arc sec}} \qquad \text{(A-26)}$$

The error sensitivity of the linear velocity error term in Equation A-12 evaluated at t = 45 min and expressed in units of (N.mi./hr)/arc seconds is as follows: (λ is assumed to be 35 deg and ψ is assumed to be 90 deg).

$$\frac{E_v'}{E_6'} = R\,\Omega^2(\cos 35°)^2(\sin 90°)^2(45)(60)$$

$$= \frac{(2.09\text{x}10^{7})(4.85\text{x}10^{-6})(3{,}600)(15)^2(4.85\text{x}10^{-6})^2(.819)^2(45)(60)}{6{,}000}$$

$$= .0006\ \frac{\text{n.mi./hr}}{\text{arc sec}} \qquad \text{(A-27)}$$

A2.8 <u>Accelerometer Scale Factor, Level Accelerometer</u>

A level accelerometer scale factor error will cause attitude, velocity and position errors that are a function of IMU velocity time history. For the special case of an acceleration impulse at the beginning of the navigation period and constant velocity (magnitude and direction) thereafter, the attitude, velocity and position errors are as follows:

$$E_\phi = \frac{E_7 V \sin \omega_0 t}{\omega_0 R} \qquad \text{(A-28)}$$

$$E_v = E_7 V \cos \omega_o t \qquad \text{(A-29)}$$

$$E_p = \frac{E_7 V}{\omega_o} \sin \omega_o t \qquad \text{(A-30)}$$

where: E_7 = accelerometer scale factor error, dimensionless

V = velocity increment, ft/sec

In the forthcoming tests, the navigation errors caused by accelerometer scale factor errors will be much less than those caused by other error sources.

A2.9 Accelerometer Misalignment, Level Accelerometers

Misalignment of the level accelerometers with respect to the level gyros in the level plane will cause velocity errors that are a function of velocity time history. These errors are similar to the accelerometer scale factor errors discussed in Section A2.8.

Misalignment of the level accelerometers with respect to the level gyros in the vergical plane will cause earth rate transformation errors. The effects of these errors are discussed in Section A2.7.

A2.10 Vertical Velocity and Position Errors

Vertical velocity and position errors will be limited by altitude damping in the RSP. It is assumed that the gain constants a_r and a_v will be set to -1 and 0, respectively. With these values, the vertical position error will be limited to the difference between the actual altitude and the altitude input, and the vertical velocity error will be small.

A2.11 Navigation Errors of a Dodecahedron

The navigation errors of a non-redundant strapdown IMU were discussed in Sections A2.2 through A2.10. This discussion will now be extended to a dodecahedron.

For the attitude sensors (SARs) of the BB DDH,

$$\underline{m} = \underline{Ab} + \underline{\varepsilon} \qquad \text{(A-31)}$$

where: $\underline{m}$ = sensor output (6 vector), rad/sec

$\underline{A}$ = transformation matrix (6x3 matrix), dimensionless

$\underline{b}$ = sensed quantity (3 vector), rad/sec

$\underline{\varepsilon}$ = sensor error (6 vector), rad/sec

$$\hat{\underline{b}} = \underline{B}\ \underline{m} \tag{A-32}$$

where: $\hat{\underline{b}}$ = least squares estimate of the quantity (3 vector), rad/sec

$$\underline{B} = (\underline{A}^T\underline{A})^{-1}\ \underline{A}^T \tag{A-33}$$

$$\tilde{\underline{b}} = \hat{\underline{b}} - \underline{b} \tag{A-34}$$

where: $\tilde{\underline{b}}$ = error in the estimate of the sensed quanity (3 vector), rad/sec

$$\tilde{\underline{b}} = \underline{B}\ \underline{\varepsilon} \tag{A-35}$$

The sensor error vector may be written as follows:

$$\underline{\varepsilon} = \underline{C}_o - \underline{M}_s\underline{W}_1\underline{a} + \underline{M}_I W_2\underline{a} + \underline{C}_1\underline{W}_3\underline{b}$$

$$+ \underline{C}_2\underline{W}_4\underline{b} + \underline{C}_3\underline{W}_5\underline{b} \tag{A-36}$$

where: $\underline{C}_o$ = bias error (6 vector), rad/sec

$\underline{M}_s$ = spin axis mass unbalance (6x6 diagonal matrix), rad/sec/g

$\underline{M}_I$ = input axis mass unbalance (6x6 diagonal matrix), rad/sec/g

$\underline{C}_1$, $\underline{C}_2$, $\underline{C}_3$ = sensor misalignment components (6x6 diagonal matrices,), rad

$\underline{W}_1$, $\underline{W}_2$, $\underline{W}_3$, $\underline{W}_4$, $\underline{W}_5$ = transformation matrices (6x3 matrices), dimensionless

$\underline{a}$ = thrust acceleration (3 vector), g

$\underline{b}$ = angular velocity (3 vector), rad/sec

Assume that the level sensed acceleration components and the level angular velocity components are negligible.

$$\underline{a} = \begin{bmatrix} a_x \\ 0 \\ 0 \end{bmatrix} \quad \underline{b} = \begin{bmatrix} b_x \\ 0 \\ 0 \end{bmatrix} \tag{A-37}$$

Equation A-36 may be rewritten as follows:

$$\underline{\varepsilon} = \underline{C}_o - \underline{M}_S \underline{W}_1' a_x + \underline{M}_I \underline{W}_2' a_x + \underline{C}_1 \underline{W}_3' b_x \tag{A-38}$$

where: $\underline{W}_1'$, $\underline{W}_2'$, $\underline{W}_3'$ = transformation vectors (6x1 vectors), dimensionless

$$\underline{W}_1' = \begin{bmatrix} S \\ S \\ C \\ C \\ 0 \\ 0 \end{bmatrix} \qquad \underline{W}_2' = -\begin{bmatrix} C \sin \beta_A \\ C \sin \beta_B \\ S \sin \beta_C \\ S \sin \beta_D \\ \sin \beta_E \\ \sin \beta_F \end{bmatrix} \qquad \underline{W}_3' = -\begin{bmatrix} C \\ C \\ S \\ S \\ 1 \\ 1 \end{bmatrix} \tag{A-39}$$

where: S = $\sin \alpha$

C = $\cos \alpha$

α = .55 rad

β_i = gimbal angle of the ith SAR (the SAR gimbal angle is measured from the position in which the OA is in the plane of the IA and the DDH X axis and is in the positive X direction), deg

The sensor misalignment matrix is

$$\underline{C}_1 = \begin{bmatrix} C_{1A} & 0 & 0 & 0 & 0 & 0 \\ 0 & C_{1B} & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{1C} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{1D} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{1E} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{1F} \end{bmatrix} \tag{A-40}$$

where: $C_{1i} = l_{0i} + l_{1i} \sin \beta_i$

l_{0i} = external misalignment of the ith SAR, rad

l_{1i} = internal misalignment of the ith SAR, rad

With no failures, the transformation matrix in Equation A-31 is

$$\underline{A} = \begin{bmatrix} S & 0 & C \\ S & 0 & -C \\ C & -S & 0 \\ C & S & 0 \\ 0 & C & S \\ 0 & C & -S \end{bmatrix} \tag{A-41}$$

The B matrix (Equation A-32) is thus

$$\underline{B} = (\underline{A}^T\underline{A})^{-1}\ \underline{A}^T$$

$$= \frac{1}{2} \begin{bmatrix} S & S & C & C & 0 & 0 \\ 0 & 0 & -S & S & C & C \\ C & -C & 0 & 0 & S & -S \end{bmatrix} \tag{A-42}$$

Assume now that the uncompensated SAR biases are statistically independent with zero mean and variance equal to $\sigma_B^{\ 2}$. Thus

$$\sigma^2_{B_A} = \sigma^2_{B_B} = \text{-----} = \sigma^2_{B_F} = \sigma^2_B \tag{A-43}$$

2

Considering only the bias term in Equation A-38, the sensor error covariance matrix is

$$\langle \underline{\varepsilon}\, \underline{\varepsilon}^T \rangle = \sigma_B^2\, \underline{I}_6$$

where: $\underline{I}_6$ = 6x6 identity matrix.

The covariance matrix of the error in the estimate of the sensed quantity is

$$\begin{aligned}
\langle \tilde{\underline{b}}\, \tilde{\underline{b}}^T \rangle &= \langle \underline{B}\, \underline{\varepsilon}\, \underline{\varepsilon}^T\, \underline{B}^T \rangle \\
&= \langle (\underline{A}^T \underline{A})^{-1}\, \underline{A}^T\, \underline{\varepsilon}\, \underline{\varepsilon}^T\, \underline{A} (\underline{A}^T \underline{A})^{-1} \rangle \\
&= (\underline{A}^T \underline{A})^{-1}\, \sigma_B^2\, \underline{I}_3 \\
&= \frac{\sigma_B^2}{2}\, \underline{I}_3
\end{aligned}$$

or

$$\begin{bmatrix} \sigma_{b_x} \\ \sigma_{b_y} \\ \sigma_{b_z} \end{bmatrix} = \sigma_B \begin{bmatrix} .707 \\ .707 \\ .707 \end{bmatrix} \tag{A-44}$$

where: $\sigma_{b_i}^2$ = variance of the ith component of the error in the estimate of the sensed quantity.

Similarly, for the spin axis mass unbalance term,

$$\langle \underline{\varepsilon}\, \underline{\varepsilon}^T \rangle = \sigma_s^2\, a_x^2 \begin{bmatrix} S^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & S^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & C^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & C^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where: σ_s^2 = spin axis mass unbalance variance

$$\langle \tilde{\underline{b}}\, \tilde{\underline{b}}^T \rangle = \frac{\sigma_s^2\, a_x^2}{4} \begin{bmatrix} 2(S^4 + C^4) & 0 & 0 \\ 0 & 2S^2C^2 & 0 \\ 0 & 0 & 2S^2C^2 \end{bmatrix}$$

and

$$\begin{bmatrix} \sigma_{b_x} \\ \sigma_{b_y} \\ \sigma_{b_z} \end{bmatrix} = \sigma_s\, a_x \begin{bmatrix} .548 \\ .316 \\ .316 \end{bmatrix} \tag{A-45}$$

For the input axis mass unbalance term with

$$\beta_A = \beta_B = \text{---} = \beta_F = 90 \text{ deg} \tag{A-46}$$

$$\langle \underline{\varepsilon}\, \underline{\varepsilon}^T \rangle = \sigma_I^2\, a_x^2 \begin{bmatrix} C^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & C^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & S^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & S^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where: σ_I^2 = input axis mass unbalance

$$\langle \underline{\tilde{b}}\ \underline{\tilde{b}}^T \rangle = \frac{\sigma_I^2\, a_x^2}{4} \begin{bmatrix} 4S^2C^2 & 0 & 0 \\ 0 & 2(S^4 + C^4) & 0 \\ 0 & 0 & 2(S^4 + C^4) \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{b_x} \\ \sigma_{b_y} \\ \sigma_{b_z} \end{bmatrix} = \sigma_I\, a_x \begin{bmatrix} .447 \\ .548 \\ .548 \end{bmatrix} \qquad \text{(A-47)}$$

For the SAR misalignment term, and ignoring the internal misalignments

$$\begin{bmatrix} \sigma_{b_x} \\ \sigma_{b_y} \\ \sigma_{b_z} \end{bmatrix} = \sigma_c\, b_x \begin{bmatrix} .447 \\ .548 \\ .548 \end{bmatrix} \qquad \text{(A-48)}$$

A3.0 FAILURE DETECTION, DIAGNOSIS AND CORRECTION

A failure of one or more sensors will cause system performance to be degraded. This degradation will be discussed in terms of the following effects:

a. Undetected errors
b. Transient errors
c. Degradation resulting from the deletion of sensors.

Undetected errors are failure induced errors that do not exceed the detection threshold. Transient errors are errors that develop in the interval from the occurrence of the failure to its detection and correction. The effects of transient errors may remain after the failure is cleared. The degradation resulting from the deletion of sensors refers to the increased error resulting from the deletion of sensors in the estimation equations.

Undetected errors and transient errors are both a function of the filter gain and time constant values in the FDDC subroutine of the RSP.

The effects of undetected errors may be evaluated with the equations in Section A2.0 and the FDDC text matrix (C matrix) in Reference 3. The effects of transient errors may be evaluated with the RSP in the Simulation Mode. (The effects of undetected errors may also be evaluated in the Simulation Mode).

The performance degradation resulting from the deletion of sensors may be evaluated with the equations in Section A2.11. The effect of one or more sensor failures may be determined by setting the appropriate row (or rows) in the A matrix (Equation A-41) equal to zero.

A4.0 ESTIMATED PERFORMANCE

Navigation accuracy in the forthcoming tests may be estimated from the equations in Section A2.0 and the estimated performance characteristics of the BB DDH. The latter information was supplied by MSFC and is summarized in Table A-1.

The navigation error estimates will be helpful in determining whether or not particular RSP changes are required. These estimates will also be helpful in designing the laboratory and van tests and in evaluating the results of these tests.

Five cases will be considered:

a. DDH not rotated during the navigation period, no prenavigation gyro drift update, gyro compensation not a function of SAR gimbal angle.

b. DDH not rotated during the navigation period, gyro drift updated prior to navigation, gyro compensation not a function of SAR gimbal angle.

c. DDH rotated about the vertical axis at the beginning of the navigation period, no prenavigation gyro drift update, gyro compensation not a function of SAR gimbal angle.

d. DDH rotated about the vertical axis at the beginning of the navigation period, gyro drift updated prior to navigation, gyro compensation not a function of SAR gimbal angle.

(no external errors or meas. errors) pretty idealized

From TRW

Table A-1
BB DDH Performance Characteristics

Error Source	Estimated Magnitude		100	240	1000
Gyro Bias	he used maximum when needed	1σ			
Day to day stability	.10 deg/hr		.012	.005	.0012
Single run stability	.02 deg/hr				
Gyro Spin Axis Mass Unbalance	shows quantization not negligible.				
Day to day stability	.05 deg/hr/g		.02	.0083	.002
Single run stability	.01 deg/hr/g				
Gyro Input Axis Mass Unbalance					
Magnitude	.10 deg/hr/g				
Day to day stability	.05 deg/hr/g				
Single run stability	.01 deg/hr/g				
Gyro Output Axis Unbalance					
Magnitude	.05 deg/hr/g	not im.			
Day to day stability	.02 deg/hr/g	X			
Single run stability	.01 deg/hr/g				
Gyro Misalignment			independent of time		
Magnitude (internal misalignment)	30 sec		.33 sec	.33	.33
Stability (total misalignment)	15 sec				
Accelerometer Bias					
Day to day stability	100 μg	47 (100)	11	4.6	1.1
Single run stability	Negl.	19 (240)			
Accelerometer Misalignment					
Stability	15 arc sec	10 (100) / 4 (240)	4.0	1.7	.48

Quantization ppm

gyro Scale Factor ppm — $< 30 \times 10^{-6}$ g/m → 1.6 | 1.6 | 1.6

accel " — ppm same as μg → 20 | 8.3 | 2.0

time independent

e. DDh rotated about the vertical axis at the beginning of the navigation period, gyro drift updated prior to navigation, gyro mass unbalance and misalignment compensation both a function of SAR gimbal angle.

Only the nonperiodic error components will be considered in the estimates. Dynamic errors and the performance degradation caused by simulated failures will not be considered.

CASE A

DDH not rotated during the navigation period, no prenavigation gyro drift update, gyro compensation not a function of SAR gimbal angle.

1. Azimuth alignment

   Error contribution negligible.

2. Level alignment

   $(0025)(10\times10^{-6})(2.06\times10^{5}) \oplus (.0006)(100\times10^{-6})(2.06\times10^{5}) \approx .05$ n.mi./hr

3. North axis attitude drift

   $60.8[(.10)(.707) \oplus (.05)(.316) \oplus (.02)(.548)] = 4.45$ n.mi./hr

4. East axis attitude drift

   $60.8[(.02)(.707) \oplus (.01)(.316) \oplus (.01)(.548)] = .94$ n.mi./hr

5. Vertical axis attitude drift

   $9.7[(.10)(.707) \oplus (.05)(.548) \oplus (.02)(.447)] = .74$ n.mi./hr

6. Gyro misalignment

   $(.0025)(15)(.548) \oplus (.0006)(15)(.447) \approx .02$ n.mi./hr

7. Accelerometer bias

   Error contribution negligible.

8. Accelerometer scale factor

   No nonperiodic error term.

9. Accelerometer misalignment

$(.0025)(15) \oplus (.0006)(15) \approx .04$ n.mi./hr

10. Total error

$.05 \oplus 4.45 \oplus .94 \oplus .74 \oplus .02 \oplus .04 \approx 4.6$ n.mi./hr

<u>CASE B</u>

DDH not rotated during the navigation period, gyro drift updated prior to navigation, gyro compensation not a function of SAR gimbal angle.

1. Azimuth alignment

Error contribution negligible.

2. Level alignment

Error contribution negligible.

3. North axis attitude drift

$60.8[(.02)(.707) \oplus (.01)(.316) \oplus (.01)(.548)] = .94$ n.mi./hr

4. East axis attitude drift

$60.8[(.02)(.707) \oplus (.01)(.316) \oplus (.01)(.548)] = .94$ n.mi./hr

5. Vertical axis attitude drift

$9.7[(.02)(.707) \oplus (.01)(.548) \oplus (.01)(.447)] = .15$ n.mi./hr

6. Gyro misalignment

Error contribution negligible.

7. Accelerometer bias

Error contribution negligible.

8. Accelerometer scale factor

No nonperiodic component.

9. Accelerometer misalignment

Error contribution negligible.

10. Total error

$.94 \oplus .94 \oplus .15 \approx 1.3$ n.mi./hr

CASE C

DDH rotated about the vertical axis at the beginning of the navigation period, no prenavigation gyro drift update, gyro compensation not a function of SAR gimbal angle.

1. Azimuth alignment

   $60.8[(.10)(.707) \oplus (.05)(.316) \oplus (.02)(.548)] = 4.45$ n.mi./hr

2. Level alignment

   $(.0025)(100\times10^{-6})(2.06\times10^{5}) \oplus (.0006)(100\times10^{-6})$
   $(2.06\times10^{5}) \approx .05$ n.mi./hr

3. North axis attitude drift

   $60.8[(.10)(.707) \oplus (.05)(.316) \oplus (.02)(.548) \oplus$
   $(.10)(.276)] = 4.78$ n.mi./hr

4. East axis attitude drift

   $60.8[(.10)(.707) \oplus (.05)(.316) \oplus (.02)(.548) \oplus$
   $(.10)(.724)] = 6.25$ n.mi./hr

5. Vertical axis attitude drift

   $9.7[(.10)(.707) \oplus (.05)(.548) \oplus (.02)(.447) \oplus$
   $(.10)(.90)] = 1.14$ n.mi./hr

6. Gyro misalignment (assume 360 deg rotation)

   $.106[(30)(.276) \oplus (30)(.724)] \approx 2.46$ n.mi.

   $.0036[(15)(.447) \oplus (30)(.90)] \oplus .0025[(1.73)(15)(.548)$
   $\oplus (30)(.276) \oplus (30)(.724)] \oplus .0006[(15)(.548) \oplus (15)(.447)$
   $\oplus (.707)(30)(.276) \oplus (.707)(30)(.724)] \approx .12$ n.mi./hr

7. Accelerometer bias

   Error contribution negligible.

8. Accelerometer scale factor

   No nonperiodic component.

9. Accelerometer misalignment

   $.0025[15 \oplus 15] \oplus .0006[15] = .05$ n.mi./hr

10. Total error

2.5 n.mi

$4.45 \oplus .05 \oplus 4.78 \oplus 6.25 \oplus 1.14 \oplus .12 \oplus .05 \approx 9.1$ n.mi./hr

CASE D

DDH rotated about the vertical axis at the beginning of the navigation period, gyro drift updated prior to navigation, gyro compensation not a function of SAR gimbal angle.

1. Azimuth alignment (assume 3 min misalignment)

   $60.8)[(15)(.819)(3)(60)(4.85\text{x}10^{-6}) \oplus (.02)(.707) \oplus (.01)(.316) \oplus (.01)(.548)] = 1.14$ n.mi./hr

2. Level alignment

   Error contribution negligible.

3. North axis attitude drift

   $60.8[(.02)(.707) \oplus (.01)(.316) \oplus (.01)(.548) \oplus (.10)(.276)] = 1.92$ n.mi./hr

4. East axis attitude drift

   $60.8[(15)(.819)(3)(60)(4.85\text{x}10^{-6}) \oplus (.02)(.707) \oplus (.01)(.316) \oplus (.01)(.548) \oplus (.10)(.724)] = 4.54$ n.mi./hr

5. Vertical axis attitude drift

   $9.7[(.02)(.707) \oplus (.01)(.548) \oplus (.01)(.447) \oplus (.10)(.90)] = .88$ n.mi./hr

6. Gyro misalignment (assume 360 deg rotation)

   $.106[(30)(.276) \oplus (30)(.724)] \approx 2.46$ n.mi

   $.0036[(15)(.447) \oplus (30)(.90)] \oplus .0025[(1.41)(15)(.548) \oplus (30)(.276) \oplus (30)(.724)] \oplus .0006[(15)(.548) \oplus (.707)(30)(.276) \oplus (.707)(30)(.724)] \approx .12$ n.mi./hr

7. Accelerometer bias

   Error contribution negligible.

8. Accelerometer scale factor

No non-periodic component.

9. Accelerometer misalignment

$(.0006)(15) = .01$ n.mi./hr

10. Total error

2.5 n.mi.

$1.14 \oplus 1.92 \oplus 4.54 \oplus .88 \oplus .12 \oplus .01 \approx 5.1$ n.mi./hr

<u>CASE E</u>

DDH rotated about the vertical axis at the beginning of the navigation period, gyro drift updated prior to navigation, gyro mass unbalance and misalignment compensation both a function of SAR gimbal angle.

1. Azimuth alignment (assume 3 min misalignment)

$60.8[(15)(.819)(3)(60)(4.85\text{x}10^{-6}) \oplus (.02)(.707) \oplus$

$(.01)(.316) \oplus (.01)(.548)] = 1.14$ n.mi./hr

2. Level alignment

Error contribution negligible.

3. North axis attitude drift

$60.8[(.02)(.707) \oplus (.01)(.316) \oplus (.01)(.548)] = .93$ n.mi./hr

4. East axis attitude drift

$60.8[(15)(.819)(3)(60)(4.85\text{x}10^{-6}) \oplus (.02)(.707) \oplus$

$(.01)(.316) \oplus (.01)(.548)] = 1.14$ n.mi./hr

5. Vertical axis attitude drift

$9.7[(.02)(.707) \oplus (.01)(.548) \oplus (.01)(.447)] = .15$ n.mi./hr

6. Gyro misalignment

$.106[(15)(.276) \oplus (15)(.724)]$ 1.23 n.mi.

$.0036[(15)(.447)] \oplus .0025[(1.41)(15)(.548)] \oplus$

$.0006[(15)(.548)] \approx .04$ n.mi./hr

7. Accelerometer bias

   Error contribution negligible.

8. Accelerometer scale factor

   No nonperiodic component.

9. Accelerometer misalignment

   $(.0006)(15) = .01$ n.mi./hr

10. Total error

    1.2 n.mi.

    $1.14 \oplus .94 \oplus 1.14 \oplus .15 \oplus .04 \oplus .01 \approx 1.9$ n.mi./hr

APPENDIX B

B1. NOMINAL NAVIGATION RUN

The following tables contain the output word list and 6-sec printouts from a test case dated 25 May 1969.

OUTPUT FROM RSSSTP

410	15	T	861	15	TG	510	15	TP	407	23	NPHS	939	23	RATE	379	15	DLPN
835	8	AHTS	836	8		837	8		838	1	WHTS	839	1		840	1	
725	8	AHTA	726	8		727	8		728	8		729	8		730	8	
733	1	OHTG	734	1		735	1		736	1		737	1		738	1	
745	-5	PHIX	746	-5	1,2	747	-5	1,3	748	-5	2,1	749	-5	2,2	750	-5	2,3
751	-5	3,1	752	-5	3,2	753	-5	3,3	866	-5	DALH	867	-5		868	-5	
733	-4	OHTG	734	-4		735	-4		736	-4		737	-4		738	-4	
700	9	DVSP	701	9		702	9		838	-4	WHTS	839	-4		840	-4	
240	3	PHID	243	3	THTL	424	3	PHIC	575	3	THTI	577	13	V	425	25	R
238	3	PHTD	247	3	TLHT	687	3	PHTC	686	3	TIHT	792	13	VRHT	681	25	RHAT
810	3	DLPD	811	3	DLTL	809	3	DLPC	831	25	DLRN	816	13	DELV	830	25	DLRE
556	1	KMTX	557	1	1,2	558	1	1,3	559	1	2,1	560	1	2,2	561	1	2,3
716	1	AHTX	717	1	1,2	718	1	1,3	719	1	2,1	720	1	2,2	721	1	2,3
562	1	K3,1	563	1	3,2	564	1	3,3	832	3	TTAV	833	3		834	3	
722	1	AH31	723	1	3,2	724	1	3,3	803	3	THTV	804	3		805	3	
435	1	WN	244	-11	WC	245	-10	WL	478	-16	WNDT	473	-16	WCDT	467	-15	WLDT
481	9	R2DV	482	9		483	9		462	6	GV	463	0	GE	464	0	GN
891	7	ALSV	892	7		893	7		504	8	ASNV	505	8		506	8	
474	-11	WV	475	-11	WE	476	-11	WNCP	545	6	GVP	543	0	GEP	544	0	GNP
519	4	DVSV	520	4		521	4		241	25	ALT	242	13	RDOT	472	9	R2DT
841	15	APUL	842	15		843	15		844	15		845	15		846	15	
847	15	GPUL	848	15		849	15		850	15		851	15		852	15	
532	1	WBV	533	1		534	1		536	1	WBLV	537	1		538	1	
540	13	VV	541	13	VE	542	13	VN	579	13	VLV	580	13		581	13	
523	-4	DALV	524	-4		525	-4		946	-4	DELH	535	1	HDOT	578	3	GMMA
513	4	DVAV	514	4		515	4		516	4		517	4		518	4	
526	-4	DAGV	527	-4		528	-4		529	-4		530	-4		531	-4	
915	23	FSG	916	23	FVGI	917	23	FFDG	918	23	FSA	919	23	FVAI	920	23	FFDA
1427	3	WR	469	-12	WVP	470	-12	WEP	471	-12	WNP	448	0	CC	449	0	SC
455	0	M1N	1434	3	THTA	1432	0	S	1432	0	C	450	0	M3	453	-1	M6
495	1	AMTX	496	1		497	1		498	1		499	1		500	1	
501	1		502	1		503	1		566	0	SMTX	567	0		568	0	
569	0		570	0		571	0		572	0		573	0		574	0	
485	0	DMTX	486	0	1,2	487	0	1,3	488	0	2,1	489	0	2,2	490	0	2,3
491	0	3,1	492	0	3,2	493	0	3,3	456	-4	ALPR	457	-4		458	-4	
688	13	VHTX	689	13	VHTY	690	13	VHTZ	697	6	GHTX	698	6	GHTY	699	6	GHTZ
694	1	URHT	695	1		696	1		767	-2	K9	762	1	K11	763	1	K33
759	2	G1V	760	2		761	2		756	9	DIGV	757	9		758	9	
691	25	RHTX	692	25	RHTY	693	25	RHTZ	713	14	DRV	714	14		715	14	

OUTPUT FROM RSSSTP

789	13	PHTV	790	13		791	13		793	13	VEHT	794	13		795	13	
806	-2	THTP	807	-2		808	-2		812	25	DLAL	817	3	DGAM	755	1	E4
900	10	AHAC	901	10		902	10		797	9	ACCH	798	9		799	9	
813	13	DLTV	814	13		815	13		921	6	GHTA	922	6		923	6	
824	1	DWBL	825	1		826	1		827	9	DELA	828	9		829	9	
903	9	DLAC	904	9		905	9		906	8	DLAS	907	8		908	8	
909	7	DPAS	910	7		911	7		912	9	DALS	913	9		914	9	
678	-5	E1	679	-5	E3	754	-5	E13	768	6	GOV	769	6		770	6	
925	9	DVSP	926	9		927	9		869	25	RSLH	870	-24	J15	539	25	RSL
682	15	TNCP	410	15	T	732	15	TE	821	6	DLGV	822	6		823	6	

NOMINAL RUN

1	15	DLPN	3600.000
3	-4	DLTN	0.4000000E-01
0	0	END	0.0000000E 00

NOMINAL RUN

1.000000	0.0000000E 00	0.9960937E 00	1.000000	1.000000	3600.000
0.0000000E 00	0.0000000E 00	-32.17920	0.0000000E 00	-0.1727581E-02	-0.1564026E-02
-27.37247	-27.37247	0.0000000E 00	0.0000000E 00	-16.91882	-16.91882
-0.1357555E-02	-0.1357555E-02	-0.7758141E-03	-0.7758141E-03	-0.2327204E-02	0.7758141E-03
0.0000000E 00	-0.1955777E-05	0.2160668E-05	0.1955777E-05	0.0000000E 00	0.0000000E 00
-0.2160668E-05	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.2160668E-05	-0.1955777E-05
-0.4242361E-04	-0.4242361E-04	-0.2424419E-04	-0.2424419E-04	-0.7272512E-04	0.2424419E-04
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.5398691E-04	-0.4887581E-04
0.7393541E 00	5.042400	0.7360153E 00	5.042473	0.0000000E 00	0.2089403E 08
0.7393599E 00	5.042401	0.7360172E 00	0.0000000E 00	0.0000000E 00	0.2089403E 08
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.9460144E 00	0.3241212E 00	0.0000000E 00	0.2183971E 00	-0.6374369E 00	-0.7389009E 00
0.9460385E 00	0.3240542E 00	0.0000000E 00	0.2183530E 00	-0.6374550E 00	-0.7388997E 00
-0.2394936E 00	0.6990116E 00	-0.6738129E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
-0.2394438E 00	0.6990275E 00	-0.6738153E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
-0.6097412E-01	0.0000000E 00	0.5523682E-01	-32.22550	0.0000000E 00	-0.5213475E-01
32.16454	0.0000000E 00	0.1073761E 00	0.0000000E 00	0.1525879E-03	-32.16470
0.4895509E-04	0.0000000E 00	0.5404575E-04	-32.16447	0.0000000E 00	-0.1074165E 00
0.0000000E 00	0.5722046E-05	-1.286589	0.0000000E 00	0.0000000E 00	0.0000000E 00
1149.000	1149.000	2000.000	2000.000	1474.000	1474.000
1273.000	1273.000	1276.000	1276.000	1268.000	1284.000
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	-0.2153218E-05	-0.1966953E-05	0.0000000E 00	0.0000000E 00	1.570796
-1.094439	-1.094439	-0.3814697E-05	-0.3814697E-05	-0.6763935E 00	-0.6764050E 00
-0.1676381E-05	-0.1676381E-05	-0.1132488E-05	-0.1132488E-05	-0.2868474E-05	0.7972121E-06
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.7411494E 00	0.6713398E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.9999993E 00	0.0000000E 00	0.3342390E-02	0.0000000E 00	-0.9999938E 00
-0.9999938E 00	0.0000000E 00	-0.3342390E-02	0.2402223E 00	-0.7011385E 00	0.6713398E 00
0.9460151E 00	0.3241215E 00	0.0000000E 00	-0.2175956E 00	0.6350975E 00	0.7411494E 00
-0.1671314E-02	0.0000000E 00	0.4999973E 00	0.0000000E 00	0.5000000E 00	0.0000000E 00
-0.4999973E 00	0.0000000E 00	-0.1671314E-02	0.0000000E 00	0.0000000E 00	0.0000000E 00
1068.296	365.9316	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.2401717E 00	-0.7011547E 00	0.6713412E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
5018152.	-0.1464995E 08	0.1402702E 08	0.0000000E 00	0.0000000E 00	0.0000000E 00

NOMINAL RUN

0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	-0.3725290E-08	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.2089403E 08	0.4786055E-07	0.2089403E 08
0.0000000E 00	1.000000	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
2.000000	1.000000	1.996094	1.000000	1.000000	3600.000
0.0000000E 00	0.0000000E 00	-32.17920	0.0000000E 00	-0.1829624E-02	-0.1564026E-02
-27.37247	-27.37247	0.0000000E 00	0.0000000E 00	-16.91882	-16.91882
-0.1357555E-02	-0.1357555E-02	-0.9696484E-03	-0.9696484E-03	-0.2327204E-02	0.7758141E-03
0.0000000E 00	-0.1955777E-05	0.2287328E-05	0.1955777E-05	0.0000000E 00	0.0000000E 00
-0.2287328E-05	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.2287328E-05	-0.1955777E-05
0.4242361E-04	-0.4242361E-04	-0.3030151E-04	-0.3030151E-04	-0.7272512E-04	0.2424419E-04
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.5717576E-04	-0.4887581E-04
0.7393541E 00	5.042400	0.7360153E 00	5.042473	0.0000000E 00	0.2089403E 08
0.7393541E 00	5.042400	0.7360172E 00	5.042473	0.0000000E 00	0.2089403E 08
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.9460144E 00	0.3241212E 00	0.0000000E 00	0.2183971E 00	-0.6374369E 00	-0.7389009E 00
0.9460146E 00	0.3241236E 00	0.4768372E-06	0.2183998E 00	-0.6374390E 00	-0.7388997E 00
-0.2394936E 00	0.6990116E 00	-0.6738129E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
-0.2394948E 00	0.6990099E 00	-0.6738153E 00	-0.7915497E-04	-0.5054474E-04	-0.5722046E-04
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
-0.6097412E-01	0.0000000E 00	0.5523682E-01	-32.22556	0.0000000E 00	-0.5213475E-01
32.16454	0.0000000E 00	0.1073608E 00	0.0000000E 00	0.1525879E-03	-32.16470
0.4895509E-04	0.0000000E 00	0.5404575E-04	-32.16447	0.0000000E 00	-0.1074165E 00
0.0000000E 00	0.5722046E-05	-1.286589	0.0000000E 00	0.0000000E 00	0.0000000E 00
1149.000	1149.000	2000.000	2000.000	1474.000	1474.000
1273.000	1273.000	1275.000	1275.000	1268.000	1284.000
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	-0.2153218E-05	-0.1966953E-05	0.0000000E 00	0.0000000E 00	1.570796
-1.094439	-1.094439	-0.3814697E-05	-0.3814697E-05	-0.6763935E 00	-0.6764050E 00
-0.1676381E-05	-0.1676381E-05	-0.1132488E-05	-0.1132488E-05	-0.2868474E-05	0.7972121E-06
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00

NOMINAL RUN

0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.7411494E 00	0.6713398E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.9999993E 00	0.0000000E 00	0.3342390E-02	0.0000000E 00	-0.9999938E 00
-0.9999938E 00	0.0000000E 00	-0.3342390E-02	0.2402223E 00	-0.7011385E 00	0.6713398E 00
0.9460151E 00	0.3241215E 00	0.0000000E 00	-0.2175956E 00	0.6350975E 00	0.7411494E 00
-0.1671314E-02	0.0000000E 00	0.4999973E 00	0.0000000E 00	0.5000000E 00	0.0000000E 00
-0.4999973E 00	0.0000000E 00	-0.1671314E-02	0.0000000E 00	0.0000000E 00	0.0000000E 00
1068.271	366.0098	0.0000000E 00	-7.728371	22.56213	-21.67307
0.2402227E 00	-0.7011378E 00	0.6713417E 00	-0.5236292E-01	0.6563640E-01	-0.3908944E-01
0.1576424E-01	-0.4602146E-01	-0.2624226E-01	-7.728333	22.56213	-21.67303
5019220.	-0.1464958E 08	0.1402702E 08	-5.982422	17.46289	-16.72070
0.9765625E-03	0.9765625E-03	0.0000000E 00	-0.9765625E-03	0.9765625E-03	0.9765625E-03
0.1788139E-05	-0.2473593E-05	0.1579523E-05	-28.00000	-0.7853985E 00	0.1192093E-05
-0.2587891E-01	0.7849121E-01	0.0000000E 00	-0.6109619E-01	0.9765625E-03	0.5572510E-01
-0.9765625E-03	0.9765625E-03	0.9765625E-03	-7.701683	22.48423	-21.67307
0.0000000E 00	0.0000000E 00	0.3337860E-05	0.7934570E-03	0.6713867E-03	0.0000000E 00
-0.3051758E-03	0.9765625E-03	0.2441406E-03	0.0000000E 00	-0.1525879E-03	-0.1449585E-01
0.1446533E-01	0.3051758E-04	0.2746582E-03	0.1446533E-01	0.0000000E 00	0.1831055E-03
0.0000000E 00	0.0000000E 00	-0.7450581E-08	-7.744133	22.60815	-21.64684
7.702393	-22.48364	21.67297	0.2089403E 08	0.4786055E-07	0.2089403E 08
1.000000	2.000000	0.0000000E 00	-0.2136230E-03	0.1708984E-02	-0.3814697E-04
3.000000	2.000000	2.996094	1.000000	1.000000	3600.000
0.0000000E 00	-0.1373291E-01	-32.17090	0.0000000E 00	-0.1747131E-02	-0.1613617E-02
-27.37247	-27.37247	0.0000000E 00	0.0000000E 00	-16.91882	-16.88666
-0.1357555E-02	-0.1357555E-02	-0.9696484E-03	-0.9696484E-03	-0.2327204E-02	0.5817413E-03
0.0000000E 00	-0.2015382E-05	0.2183020E-05	0.2015382E-05	0.0000000E 00	0.0000000E 00
-0.2183020E-05	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.2183020E-05	-0.2015382E-05
-0.4242361E-04	-0.4242361E-04	-0.3030151E-04	-0.3030151E-04	-0.7272512E-04	0.1817942E-04
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.5459785E-04	-0.5042553E-04
0.7393541E 00	5.042400	0.7360153E 00	5.042545	0.0000000E 00	0.2089403E 08
0.7393541E 00	5.042401	0.7360172E 00	5.042546	0.0000000E 00	0.2089403E 08
0.0000000E 00	0.9536743E-06	0.0000000E 00	0.0000000E 00	0.0000000E 00	12.00000
0.9459910E 00	0.3241899E 00	0.0000000E 00	0.2184434E 00	-0.6374211E 00	-0.7389009E 00
0.9459910E 00	0.3241925E 00	0.4768372E-06	0.2184463E 00	-0.6374230E 00	-0.7388997E 00
-0.2395444E 00	0.6989942E 00	-0.6738129E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
-0.2395458E 00	0.6989925E 00	-0.6738153E 00	-0.7820129E-04	-0.6103516E-04	-0.6389618E-04
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
-0.6097412E-01	0.0000000E 00	0.5523682E-01	-32.22550	0.0000000E 00	-0.5213475E-01

NOMINAL RUN

-32.16454	0.0000000E 00	0.1073761E 00	0.0000000E 00	0.1525879E-03	-32.16470
0.4895509E-04	0.0000000E 00	0.5404575E-04	-32.16447	0.0000000E 00	-0.1074165E 00
0.0000000E 00	0.5722046E-05	-1.286589	0.0000000E 00	0.0000000E 00	0.0000000E 00
1149.000	1149.000	2000.000	2000.000	1474.000	1475.000
1273.000	1273.000	1275.000	1275.000	1268.000	1283.000
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	-0.2153218E-05	-0.1966953E-05	0.0000000E 00	0.0000000E 00	1.570796
-1.094439	-1.094439	-0.3814697E-05	-0.3814697E-05	-0.6763935E 00	-0.6764050E 00
-0.1676381E-05	-0.1676381E-05	-0.1132488E-05	-0.1132488E-05	-0.2868474E-05	0.7972121E-06
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.7411494E 00	0.6713398E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.9999993E 00	0.0000000E 00	0.3342390E-02	0.0000000E 00	-0.9999938E 00
-0.9999938E 00	0.0000000E 00	-0.3342390E-02	0.2402732E 00	-0.7011212E 00	0.6713398E 00
0.9459918E 00	0.3241901E 00	0.0000000E 00	-0.2176417E 00	0.6350818E 00	0.7411494E 00
-0.1671314E-02	0.0000000E 00	0.4999973E 00	0.0000000E 00	0.5000000E 00	0.0000000E 00
-0.4999973E 00	0.0000000E 00	-0.1671314E-02	0.0000000E 00	0.0000000E 00	0.0000000E 00
1068.244	366.0869	0.0000000E 00	-7.730019	22.56158	-21.67309
0.2402737E 00	-0.7011201E 00	0.6713414E 00	-0.5236292E-01	0.6563663E-01	-0.3908920E-01
0.1576757E-01	-0.4602051E-01	-0.2624226E-01	-7.730835	22.56128	-21.67310
5020288.	-0.1464922E 08	0.1402702E 08	-4.787109	13.96875	-13.37695
0.9765625E-03	0.0000000E 00	0.0000000E 00	0.0000000E 00	0.9765625E-03	0.0000000E 00
0.1996756E-05	-0.2443790E-05	0.1907349E-05	-20.00000	0.0000000E 00	0.9536743E-06
-0.2673340E-01	0.7812500E-01	-0.1220703E-03	-0.6109619E-01	0.6103516E-04	0.5560303E-01
0.0000000E 00	0.9765625E-03	0.0000000E 00	-7.703323	22.48369	-21.67309
0.1192093E-05	0.2384186E-06	0.7152557E-06	-0.6103516E-04	0.3051758E-03	-0.1220703E-03
-0.3051758E-03	0.6103516E-04	0.1220703E-03	0.0000000E 00	-0.1388550E-01	-0.6195068E-02
0.6103516E-02	0.3051758E-04	0.1400757E-01	0.6103516E-02	0.0000000E 00	0.1385498E-01
0.0000000E 00	0.0000000E 00	0.0000000E 00	-7.745781	22.60760	-21.64685
7.704041	-22.48309	21.67297	0.2089403E 08	0.4786055E-07	0.2089403E 08
2.000000	3.000000	0.0000000E 00	-0.2593994E-03	-0.3051758E-04	-0.3814697E-04
4.000000	3.000000	3.996094	1.000000	1.000000	3600.000
0.0000000E 00	0.0000000E 00	-32.17920	0.0000000E 00	-0.1645088E-02	-0.1613617E-02
-27.37247	-27.37247	0.0000000E 00	0.0000000E 00	-16.91882	-16.91882
-0.1357555E-02	-0.1357555E-02	-0.7758141E-03	-0.7758141E-03	-0.2327204E-02	0.5817413E-03
0.0000000E 00	-0.2015382E-05	0.2056360E-05	0.2015382E-05	0.0000000E 00	0.0000000E 00
-0.2056360E-05	0.0000000E 00	0.0000000E 00	0.0000000E 00	-0.2056360E-05	-0.2015382E-05

B2. ALIGN RUN

The following tables contain the output word list and 3-sec printouts from a test case dated 28 May 1969.

```
OUTPUT FROM RSSSTP
 410   15    T  861   15   TG 510   15   TP  407   23 NPHS  409   23 TIME 379   15 4LP5
 975   -3 CHTX  976   -3       977   -3        978   -3        979   -3       980   -3
 981   -3       982   -3       983   -3       1017    6 DVVA1018    6      1019    6
1026   -9 ALFE1027   -9      1028   -9      1023    7 VHTA1024    7      1025    7
1034    6   Y1 1035    6 Y2  1036    0 X1   1037    0 X2  1038    0 X3    939   23 LRFG
1042   -3 SQJ11043   -3 SQJ21044   -3 SQJ31011   -4 WHS11012   -4      1013   -4
 832    3 TTAV  833    3 TTAE 834    3 TTAN  678   -5  E1   679   -5 E3    754   -5  E13

       ...IN PROGRAM INPUT
  67    3 PHI3    -1.570796
 239   23 ALGN     1.000000
 233    3 PHIH    0.9999999E-01
 234    3    2    0.9999999E-01
 235    3 PHT3    -1.470796
 250   23 NN6      1800.000
   0    0 END     0.0000000E 00

       ONE SECOND TO INITIALIZE
   1   15 DLPN     1.000000
   3   -4 DLTN    0.4000000E-01
   0    0 END     0.0000000E 00

     ...E SECOND TO INITIALIZE

  1.000000         1.000000         0.9960937E 00    1.000000         0.0000000E 00    1.000000
 0.0000000E 00    0.0000000E 00    0.0000000E 00    0.0000000E 00    0.0000000E 00    0.0000000E 00
 0.0000000E 00    0.0000000E 00    0.0000000E 00    0.0000000E 00    0.0000000E 00    0.0000000E 00
 0.0000000E 00    0.0000000E 00    0.0000000E 00    0.0000000E 00    0.0000000E 00    0.0000000E 00
 0.0000000E 00    0.0000000E 00    0.0000000E 00    0.0000000E 00    0.0000000E 00    1.000000
 0.0000000E 00    0.0000000E 00    0.0000000E 00    0.0000000E 00    0.0000000E 00    0.0000000E 00
-0.7250786E-02   -0.3317232E 00   -0.5964088E 00    0.0000000E 00    0.0000000E 00    0.3725290E-08

                 ALIGNMENT TEST
   1   15 DLPN     400.0000
   3   -4 DLTN    0.4000000E-01
   0    0 END     0.0000000E 00
```

ALIGNMENT TEST

2.000000	2.000000	0.9960937E 00	2.000000	0.0000000E 00	400.0000
0.0000000E 00	-0.5468726E-05	-0.4470348E-06	0.5468726E-05	0.0000000E 00	-0.4395843E-05
0.4470348E-06	0.4395843E-05	0.0000000E 00	-3.195099	-3.211380	-0.3202057E 00
0.5827914E-04	-0.4463363E-06	-0.4382804E-04	3.195099	3.211380	-31.84444
-3.499878	-2.876099	-0.2384186E-05	0.8940792E-01	-0.1087991E 00	1.000000
0.9999999E-01	0.1001045E-02	0.1001045E-02	0.1572073E-05	0.0000000E 00	-0.1601875E-05
0.9001827E-01	-0.9002686E-03	0.9984970E-03	-0.9915590E-02	-0.3997087E-02	0.0000000E 00

3.000000	3.000000	1.996094	2.000000	0.0000000E 00	400.0000
0.0000000E 00	-0.1192093E-06	-0.4798174E-05	0.1192093E-06	0.0000000E 00	0.1192093E-06
0.4798174E-05	-0.1192093E-06	0.0000000E 00	-0.3196716E-01	-0.3211975E-01	-0.4348755E-02
0.5371030E-04	-0.4802132E-05	-0.4907977E-04	0.3196716E-01	0.3211975E-01	-32.16029
-0.3486633E-01	-0.2895355E-01	-0.4835129E-03	0.8985996E-03	-0.1082182E-02	1.000000
0.9999855E-01	0.4315376E-04	0.4315376E-04	0.4884601E-04	0.0000000E 00	-0.5203485E-04
0.8955288E-01	-0.6675720E-05	0.9536743E-06	0.1454353E-03	0.2336502E-04	-0.1490116E-07

4.000000	4.000000	2.996094	2.000000	0.0000000E 00	400.0000
0.0000000E 00	-0.7450581E-07	-0.4827976E-05	0.7450581E-07	0.0000000E 00	0.2235174E-06
0.4827976E-05	-0.2235174E-06	0.0000000E 00	-0.1983643E-03	-0.6103516E-04	0.3967285E-03
0.5366560E-04	-0.4825182E-05	-0.4913518E-04	0.1983643E-03	0.6103516E-04	-32.16504
-0.7629395E-04	-0.1907349E-03	-0.9620190E-03	0.3099442E-05	-0.1192093E-05	1.000000
0.9955977E-01	0.0000000E 00	0.0000000E 00	0.4884601E-04	0.0000000E 00	-0.4881620E-04
0.8859444E-01	-0.7629395E-05	0.0000000E 00	0.0000000E 00	-0.4768372E-06	-0.7450581E-08

5.000000	5.000000	3.996094	2.000000	0.0000000E 00	400.0000
0.0000000E 00	-0.1639128E-06	-0.4768372E-05	0.1639128E-06	0.0000000E 00	0.2086163E-06
0.4768372E-05	-0.2086163E-06	0.0000000E 00	-0.5798340E-03	-0.1525879E-04	-0.1525879E-03
0.5367026E-04	-0.4773727E-05	-0.4913518E-04	0.2441406E-03	0.1525879E-04	-32.16504
-0.6866455E-04	-0.5798340E-03	-0.5752206E-02	0.6556511E-05	-0.7152557E-06	1.000000
0.9827963E-01	0.0000000E 00	0.0000000E 00	0.5201995E-04	0.0000000E 00	-0.4881620E-04
0.8286285E-01	-0.5722046E-05	0.0000000E 00	0.0000000E 00	-0.1645088E-04	0.3725290E-08

6.000000	6.000000	4.996094	2.000000	0.0000000E 00	400.0000
0.0000000E 00	-0.7450581E-07	-0.4470348E-05	0.7450581E-07	0.0000000E 00	0.1490116E-06
0.4470348E-05	-0.1490116E-06	0.0000000E 00	-0.1983643E-03	0.0000000E 00	0.3967285E-03
0.5369657E-04	-0.4464760E-05	-0.4913518E-04	0.1983643E-03	0.0000000E 00	-32.16504
-0.1525879E-04	-0.1983643E-03	-0.2820969E-02	0.1907349E-05	-0.1192093E-06	1.000000
0.9585029E-01	0.0000000E 00	0.0000000E 00	0.5042553E-04	0.0000000E 00	-0.4625320E-04
0.8005142E-01	-0.8583069E-05	0.0000000E 00	0.0000000E 00	-0.3814697E-05	0.0000000E 00

Appendix C

FDDC FILTER CONSTANTS

The method of deriving initial values of the FDDC prefilter gain and time constant is given here.

C1.0 FILTER DESCRIPTION

V' is a vector of 15 test signals each of the form:

$$V'_{in} = CX1 + OX2 - SY1 + OY2 - CZ1 + SZ2$$

where C and S are the sin and cosine of α, (.55 radian)

and X1, X2, Y1, Y2, Z1, Z2 are the sensor outputs.

Before comparison to the threshold value (unity), each of the 15 gyro test signals is passed through a first order low pass filter of the form:

$$Fi = \frac{Kf}{\tau_f S+1} V'_{in} \qquad \text{(C1)}$$

If V'_{in} is a step function

$$Fi = V'_{in} Kf (1-e^{-t/\tau}) \qquad \text{(C2)}$$

$$= \sin\alpha^{*} \omega e \; Kf (1-e^{-t/\tau})$$

where <u>$Fi \geq 1$ constitutes a failure state</u>

and ωe is the failed gyro error (bias)

Kf is the filter gain

τ is the filter time constant

and α is the DDH alignment angle, .55 radian.

---
* or $\cos\alpha$

C2.0 FILTER THRESHOLD

The steady state filter threshold is 1/Kf, computed as follows:

For $t \gg \tau$, equation (C2) becomes:

$$Fi = V_{in}^{'} \, Kf$$

Since Fi = 1 constitutes a failure, the threshold (for $V_{in}^{'}$) is:

$$TH_V = \frac{1}{Kf} \text{ (radian/second)} \tag{C3}$$

The failure threshold for the output of a single gyro is 1/Kf sinα, or:

$$TH_g = \frac{1.9}{Kf} \tag{C4}$$

C3.0 QUANTIZATION VERSUS GAIN AND TIME CONSTANT

For intervals short compared to the time constant

For $t \ll \tau$, equation (C2) becomes:

$$Fi = V_{in}^{'} \, Kf \left(\frac{t}{\tau}\right) \tag{C5}$$

At failure Fi = 1, therefore, the maximum rate is:

$$V_{in}^{'} \text{ (max)} = \frac{\tau}{t \; Kf} \tag{C6}$$

or, expressed in terms of error angle

$$\Theta_{(max)} = V_{in}^{'}(max)t = \frac{\tau}{Kf} = TH \; (\tau) \tag{C7}$$

for periods short compared to the time constant.

The maximum quantization as a function of filter parameters is, therefore,

$$Q_{max} = \frac{\tau}{Kf} \tag{C8}$$

Or, given the necessary filter threshold and the quantization, the minimum permissible time constant, τ_m, is:

$$\tau_m = Q\ Kf = \frac{Q}{TH} \tag{C9}$$

C4.0 COMPUTING TIME TO FAILURE

From (C2), failure is indicated when

$$V'i\ Kf\ (1-e^{-t/\tau}) = 1 \tag{C10}$$

Solving for t

$$t_f = \tau \log \left(\frac{V'i}{V'i - 1/Kf} \right) \tag{C11}$$

C5.0 COMPUTATION OF FILTER CONSTANTS

The minimum failure threshold that will not trigger on normal (3σ) instrument behavior plus noise is desired.

First an estimate must be made of the expected total uncertainty in each gyro error (fixed drift $\oplus$ g sensitive $\oplus$ misalignment error X constant rate $\oplus$ dynamic errors) under actual test conditions, after instrument compensation. Then the expected variance in the test signal $V'i$ should be computed by adding the variances of four gyros (but neglecting quantization and high frequency noise), and finally compute $\sigma V'i$, the (expected) standard deviation of the test signals.

C6.0 SELECTION OF THRESHOLD (GAIN)

Some margin must be allowed between expected instrument behavior and the failure threshold. The necessary margin is a function of the noise present and the degree of filtering. For a first assumption the initial value of the failure threshold will be set at $4\sigma V_i'$.

From (C3)

$$Kf = \frac{1}{4\sigma V'_i} \text{ (second/radian)} \tag{C12}$$

In estimating $\sigma V_i'$ it is seen that if constant high rate turns (10 degrees/second) are allowed, and a total misalignment uncertainty of 1 arc minute, 3σ is assumed, $4\sigma V_i'$ is on the order of sin (1 arc minute) X 36,000 ≈ 12 degrees/hour. Setting the threshold at this level would prohibit detection of small but intolerable bias shifts.

As an alternative, if we neglect the misalignment error in estimating $4\sigma V_i'$, a preliminary estimate is 0.2 degrees/hour or

$$TH = .2 \text{ degrees/hour} \simeq 10^{-6} \text{ radian/second} \tag{C13}$$

and

$$Kf = 10^{6} \text{ second/radian} \tag{C14}$$

C7.0 SELECTIONS OF TIME CONSTANT τ

From (C9)

$$\tau_m = Q\ Kf \tag{C15}$$

For the BB DDH the gyro quantization, $\Delta\Theta$, ≈ 2.4×10^{-5} radian and a worst case estimate of the quantization on V_i' is 7×10^{-5} radian

$$\therefore \tau_m = (7 \times 10^{-5})\ Kf = 70 \text{ seconds} \tag{C16}$$

where τ_m is the minimum time constant to avoid triggering the failure detection logic on quantization noise.

For τ five to ten times larger than τ_m, quantization noise is effectively suppressed at the expense of transient response. The initial value of τ will be set at 500 seconds, the time used in digital simulations.

C8.0 EVALUATION OF SELECTED CONSTANTS

Using the values of

$$Kf = 10^6 \text{ second/radian}$$

and

$$\tau = 500 \text{ seconds}$$

from (C7)

$$\Theta_{max} = \frac{\tau}{Kf} = \frac{500}{10^6} \text{ radian} \approx 103 \text{ arc second} \qquad \text{(C17)}$$

Again assuming a sensor misalignment angle γ, of 1 arc minute, the maximum permissible vehicle turn (in a time short relative to the time constant, e.g., < 150 seconds) would be (approximately)

$$\frac{\Theta_{max}}{\sin\alpha \ \sin\gamma} = \frac{5.10^{-4} \text{ radian}}{.00015} = 190 \text{ degrees} \qquad \text{(C18)}$$

The required threshold to allow a 360 degree turn would be .38 degrees/hour which is large relative to permissible gyro drift errors.

The selected values of $Kf = 10^6$ and $\tau = 500$ are, therefore, considered adequate for initial values, subject to the limitations discussed; turns will be limited probably to 180 degrees, by the misalignment uncertainty.

C9.0 TEST CASE RESULTS

Table C-I is a summary of results of a filter simulation using the recommended values of:

$$\tau = 500 \text{ seconds}$$

$$Kf = 10^6 \text{ second/radian}$$

$$\text{Threshold} = .2 \text{ degrees/hour}$$

and quantization of the test signal (summation of 4 gyro outputs) is 13 arc seconds.

Table C-I. Filter Simulation Results

Gyro Test Signal Error[1] °/hr [1]	Predicted Time of[2] Failure-sec	Actual[2] Time of Failure-sec	Accumulated Error Angle Arc seconds
.2	∞		∞
.25	804	725	182
.4	346	329	143
.8	144	162	117
1.6	67	66	117
3.2	32	32	104
6.4	16	17	104
10	10.1	9.3	104
12.8	7.8	8.5	104
25.6	3.9	4.4	104
250	.4	.44	104

[1] Corresponds to $\sin\alpha$ (gyro error) or .53 (gyro error)

[2] Differs from predicted time due to random quantization error

Figure C-1 is a plot of failure detection time and accumulated error angle versus V_i'/threshold, the ratio of the test signal error to the threshold.

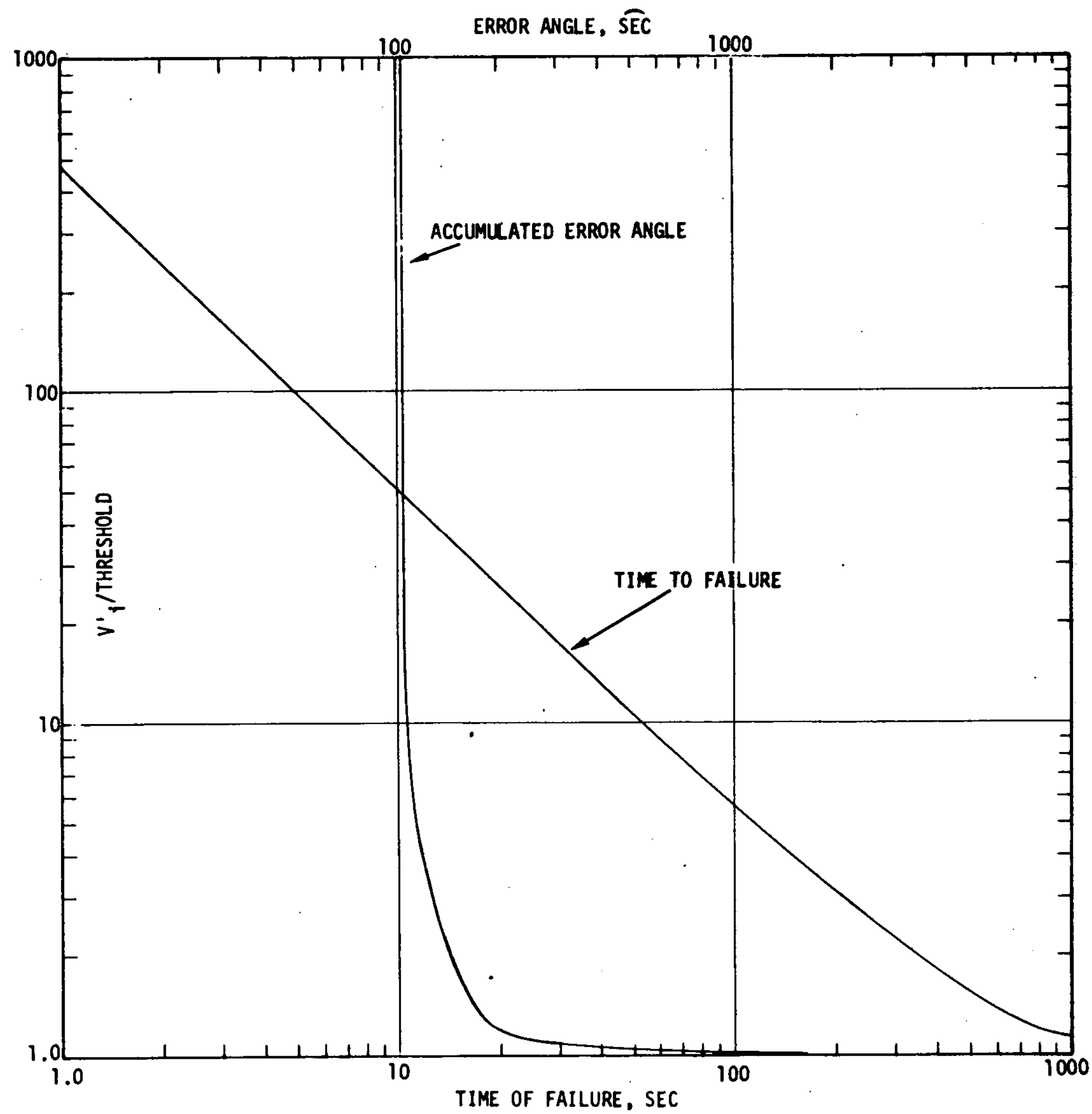


Figure C1. Failure Detection Time and Total Error Angle versus Threshold

C10.0 COMPUTATION OF DIGITAL FILTER CONSTANTS

The digital form of the prefilter is

$$F_{(i)} = Kf_1 \, F_{(i-1)} + Kf_2 \, V'_{in} \qquad \text{(C19)}$$

where V'_{in} is the test signal, (sinα ωe)

$$Kf_1 = e^{-T/\tau} \qquad \text{(C20)}$$

where T is the sample interval, .04 seconds

and

$$Kf_2 = Kf\ (1-Kf_1) \qquad \text{(C21)}$$

For τ = 500 seconds

$$Kf_{1_g} = e^{-.04/500} = .99992 \qquad \text{(C22)}$$

Substituting (C14) and (C22) in (C21)

$$Kf_{2_g} = (1-.99992)\ 10^6 = 80 \text{ seconds/radian} \qquad \text{(C23)}$$

C11.0 SUMMARY

The initial values of the FDDC prefilter are, therefore:

$$Kf_{1_g} = .99992$$

$$Kf_{2_g} = 80 \text{ seconds/radian}$$

For a filter threshold of .2 degrees/hour (on V'_i) and a time constant of 500 seconds.